Gross Lift - Off Mass Optimization for

Space Launch Vehicles

A Project report submitted in partial fulfilment of the requirement
for the award of the degree of

BACHELOR OF TECHNOLOGY
IN
MECHANICAL ENGINEERING

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SANGIVALASA, VISAKHAPATNAM (District) – 531162

2015-19

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ACKNOWLDGEMENT

We express immensely our deep sense of gratitude to **Mr.RDV.PRASAD**, Assistant Professor, Department of Mechanical Engineering, Anil Neerukonda Institute of Technology & Sciences, Sangivalasa, BheemunipatnamMandal, Visakhapatnam district for his valuable guidance and encouragement at every stage of the work made it a successful fulfillment.

We are very thankful to **Prof T.Subrahmanyam**,Principal and **Prof B.NagaRaju**, Head of the Department, Mechanical Engineering, Anil Neerukonda Institute of Technology & Sciences for their valuable suggestions.

We express our sincere thanks to the members of non-teaching staff of Mechanical Engineering for their kind co-operation and support to carry on work.

Last but not the least, we like to convey our thanks to all who havecontributed either directly or indirectly for the completion of our work.

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ABSTRACT

Rockets have multiple stages because the effectiveness of a rocket is inversely proportional to its mass and number of stages allows us to reduce the mass of the rockets as it operates. The single stage has a lot of empty fuel tank mass that we need to carry with us.The multi-stage has dropped its empty fuel tank and become a smaller; more effective rocket. The lower power stage will be flying up with empty tanks at the expense of fuel of a new tank. Each and every kilogram out will be beneficial. The masses of the empty tanks would not be nice to keep the engines and let go the tanks. We are doing a mathematical optimization of gross lift off mass for a required burnout velocity and payload ,optimal weight distribution for arbitrary number of stages like 2,3,4 and 5 having different structural ratios and specific impulses in each stage, staging optimization gives a quick insight about vehicle performance capability prior to trajectory design. Finally with this optimization technique we compared to gross lift off mass variation for two launch vehicles (i.e Ariane-1 and Proton m) for various burnout velocities and payloads at different number of stages.

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NOMENCLATURE

$\Delta v = maximum\ change\ of\ speed$

C_j= relative exhaust gas velocity

Λ=mass ratio

$m_o = initial\ mass$

$m_f = final\ mass$

$m_s = structural\ mass$

$m_p = propellent\ mass$

$m_{pl} = payload\ mass$

$I_{sp} = specific\ impulse$

$g_o = gravity\ at\ sea\ level$

$\Lambda_k = mass\ ratio$ of k^{th} stage

$\varepsilon_k = structural\ ratio$ of k^{th} stage

$\lambda_k = payload\ ratio$ of k^{th} stage

$V_{orbit} = orbital\ velocity\ at\ radial\ distance$

GM=earths gravitational parameter

r=radial distance from the Earths center to the satellite

a=semi major axis

$\Delta V_g = velocity\ loss\ due\ to\ gravity$

$\Delta V_d = velocity\ loss\ due\ to\ aerodynamic\ drag$

$\Delta V_p = propulsive\ loss\ due\ to\ steering\ and\ pressure\ change$

ΔV_{gain} = velocity gain due to Earth's rotation

P = Lagrangers multiplier

a = Semi majoraxis of an elliptical orbit

φ = Latitude angle of a location

γ = Flight path angle

α = Steering angle

A = Azimuth angle

r_p = Payload injection radius from foacal point

ω = Argument of perigee

Ω = Right ascension of the ascending node

e = Eccentricity of an elliptical orbit

i = Inclination

CHAPTER 1

1. INTRODUCTION

1 Introduction to Rockets:

The study of rockets is an excellent way for students to learn the basics of forces and the response of an object to external forces. All rockets use the thrust generated by a propulsion system to overcome the weight of the rocket. For full scale satellite launchers, the weight of the payload is only a small portion of the lift-off weight. Most of the weight of the rocket is the weight of the propellants. As the propellants are burned off during poweredascent, a larger proportion of the weight of the vehicle becomes the near-empty tankage and structure that was required when the vehicle was fully loaded.

1.1Types of Staging of Rocket:

In order to lighten the weight of the vehicle to achieve orbital velocity, most launchers discard a portion of the vehicle in a process called **staging**. There are two types of rocket staging, serial and parallel.

1.1.1Serial Staging:

In this Staging there is a small, second stage rocket that is placed on top of a larger first stage rocket. The first stage is ignited at launch and burns through the powered ascent until its propellants are exhausted. The first stage engine is then extinguished, the second stage separates from the first stage, and the second stage engine is ignited. The payload is carried atop the second stage into orbit. Serial staging was used on the Saturn V moon rockets. The Saturn V was a three stage rocket, which performed two staging maneuvers on its way to earth orbit. The discarded stages of the Saturn V were never retrieved.

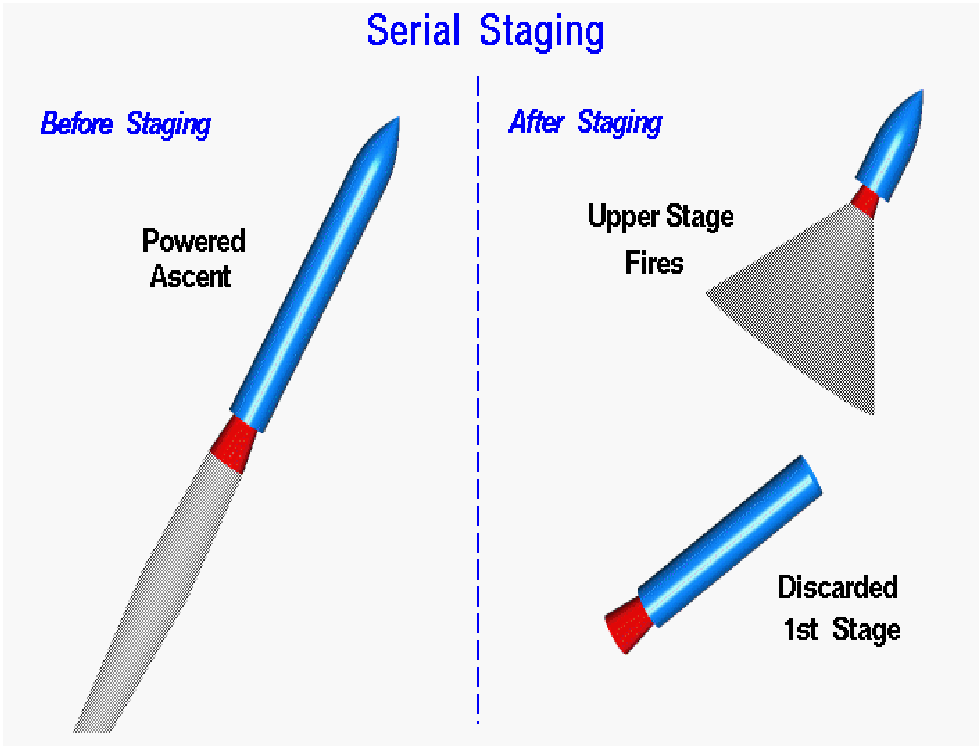

Fig 1.1 Serial Staging

1.1.2 Parallel Staging:

In this staging, as shown in the Fig 1.1, several small first stages are strapped onto to a central sustainers rocket. At launch, all of the engines are ignited. When the propellants in the strap-on are extinguished, the strap-on rockets are discarded. The sustainers engine continues burning and the payload is carried atop the sustainers rocket into orbit. Parallel staging is used on the Space Shuttle. The discarded solid rocket boosters are retrieved from the ocean, re-filled with propellant, and used again on the Shuttle. Some launchers, like the Titan III's and Delta II's, use both serial and parallel staging. The Titan III has a liquid-powered, two stage Titan II for a sustainers and two solidrocket strap-on at launch. After the solids are discarded, the sustainers engine of the Titan II burns until its fuel is exhausted. Then the second stage of the Titan II is burned, carrying the payload to orbit. The Titan III is another example of a three stage rocket.

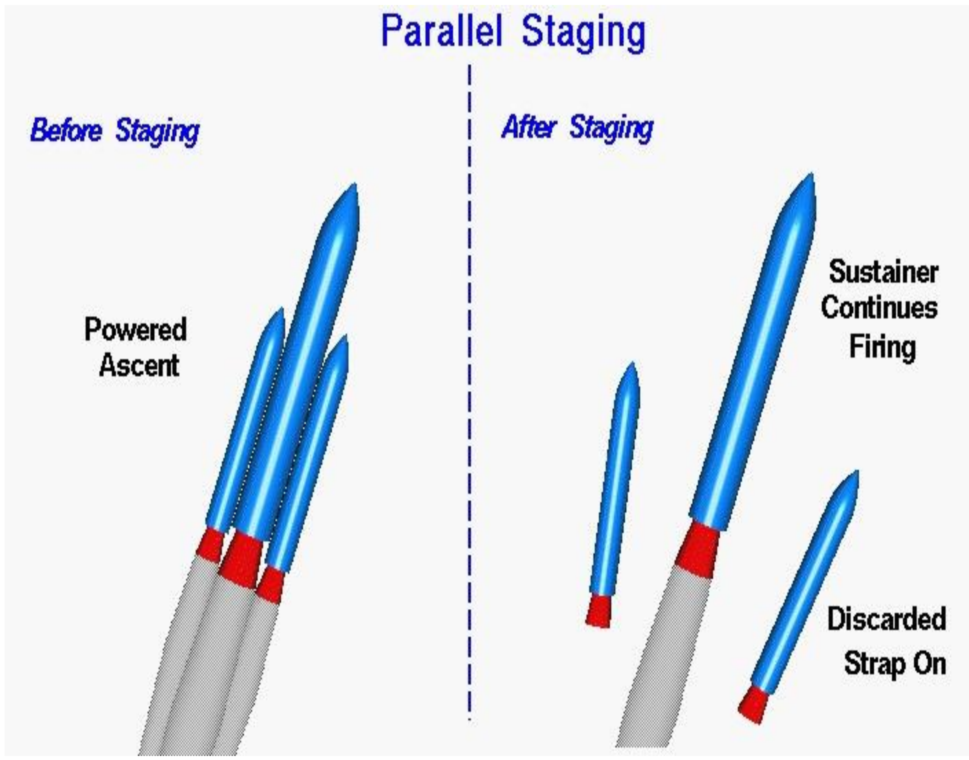

Fig 1.2 Parallel Staging

1.2 Effectiveness of a Staging

A rapid analytical method for the optimization of rocket propulsion systems is presented for a vertical take-off, horizontal landing, single-stage-to-orbit launch vehicle. This method utilizes trade-offs between propulsion characteristics affecting flight performance and engine system mass. The performance results from a point-mass trajectory optimization program are combined with a linearized sizing program to establish vehicle sizing trends caused by propulsion system variations. The linearized sizing technique was developed for the class of vehicle systems studied here. The specific examples treated in this paper are the optimization of nozzle expansion ratio and lift-off thrust-to-weight ratio to achieve either minimum gross mass or minimum dry mass. Assumed propulsion system characteristics are high chamber pressure, liquid oxygen and liquid hydrogen propellants, conventional bell nozzles, and the same fixed nozzle expansion ratio for all engines on a vehicle.The single stage rocket is not a very feasible way to place masses into orbit given the fact that the mass ratio between initial mass and burnout mass is not smaller than 0.1, and the Isp (specific impulse) of fuels is no higher than ~450 and the thrust to weight ratio is no bigger than 2 or so. This leads us to multistage rockets. multistage rocket, or step rocket, is a launch vehicle that uses two or more rocket stages, each of which contains its own engines and propellant. A tandem or serial stage is mounted on top of another stage; a parallel stage is attached alongside another stage. The result is effectively two or more rockets stacked on top of or attached next to each other. Two-stage rockets are quite common, but rockets with as many as five separate stages have been successfully launched. By jettisoning stages when they run out of propellant, the mass of the remaining rocket is decreased. Each successive stage can also be optimized for its specific operating conditions.

1.3 Payload:

It is the carrying capacity of an aircraft or launch vehicle, usually measured in terms of weight. Depending on the nature of the flight or mission, the payload of a vehicle include may cargo, passengers, flight crew, munitions, scientific instruments or experiments, or other equipment. Extra fuel, when optionally carried, is also considered part of the payload. In a commercial context (i.e., an airline or air freight carrier), payload may refer only to revenue-generating cargo or paying passengers for a rocket, the payload can be a satellite, space probe, or spacecraft carrying humans, animals, or cargo. For a ballistic missile, the payload is one or morewarheads and related systems; the total weight of these systems is referred to as the throw-weight.

The fraction of payload to the total liftoff weight of the air or spacecraft is known as the "payload fraction".When the weight of the payload and fuel are considered together, it is known as the "useful load fraction". In spacecraft, "mass fraction" is normally used, which is the ratio of payload to everything else, including the rocket structure.

1.4 Specific Impulse:

It is a measure of how effectively a rocket uses propellant or a jet engine uses fuel. By definition, it is the total impulse (or change in momentum) delivered per unit of propellant consumed and is dimensionally equivalent to the generated thrust divided by the propellant massflowrate or weight flow rate. If mass (kilogram, pound-mass, or slug) is used as the unit of propellant, then specific impulse has units of velocity. If weight (newton or pound-force) is used instead, then specific impulse has units of time (seconds). Multiplying flow rate by the standard gravity (g_0) converts specific impulse from the mass basis to the weight basis.

Specific impulse includes the contribution to impulse provided by external air that has been used for combustion and is exhausted with the spent propellant. Jet engines use outside air, and therefore have a much higher specific impulse than rocket engines. The specific impulse in terms of propellant mass spent has units of distance per time, which is a notional velocity called the *effective exhaust velocity*. This is higher than the *actual* exhaust velocity because the mass of the combustion air is not being accounted for. Actual and effective exhaust velocity are the same in rocket engines not utilizing air or other intake propellant such as water.

Specific impulse is inversely proportional to specific fuel consumption (SFC) by the relationship $I_{sp} = 1/(g_o \cdot \mathrm{SFC})$ for SFC in kg/(N·s) and $I_{sp} = 3600/\mathrm{SFC}$ for SFC in lb/(lbf·hr).

1.5 Lift Off Mass:

It is the initial mass of a Rocket. Lift off mass includes the total mass of fuels and oxidizers. Lift off mass should always be greater than the weight of the Rocket otherwise the Rocket would never takeoff. Generally a Rocket moves due to acceleration in the forward direction.While take off an upper thrust acts on the rocket. This thrust should equal the forces of gravity as well as the drag forces created by pushing the gases downwards. The sole aim of Rocket is to be propagated to a point where gravity does not act any more.

1.6 Solidboosters:

Solid fuel rocket boosters are large solid propellant motors used to provide thrust in spacecraft launches from initial launch through the first ascent stage.

1.7 Total Mass:

It constitutes of both structural and propellent mass.

1.8 Structural Ratio:

Structural ratio is defined as the ratio of structural mass to the total mass

1.9 Payload Ratio:

It is the ratio of payload mass to the total mass.

1.10 Tsiolkovsky's Rocket Equation:

$$\Delta v = v_e \ln \frac{m_0}{m_f} = I_{sp} g_0 \ln \frac{m_0}{m_f}$$

Tsiolkovsky rocket equation, classical rocket equation, or ideal rocket equation is a mathematical equation that describes the motion of vehicles that follow the basic principle of a rocket.This tells us that the change in velocity achievable is equal to the effective exhaust velocity times the natural log of the initial mass divided by the final mass. So we can see that the greater the ratio between the initial and final mass of the rocket, the more effective the rocket canbe.In the below picture are depictions of two rockets. The one on the left is a single stage.

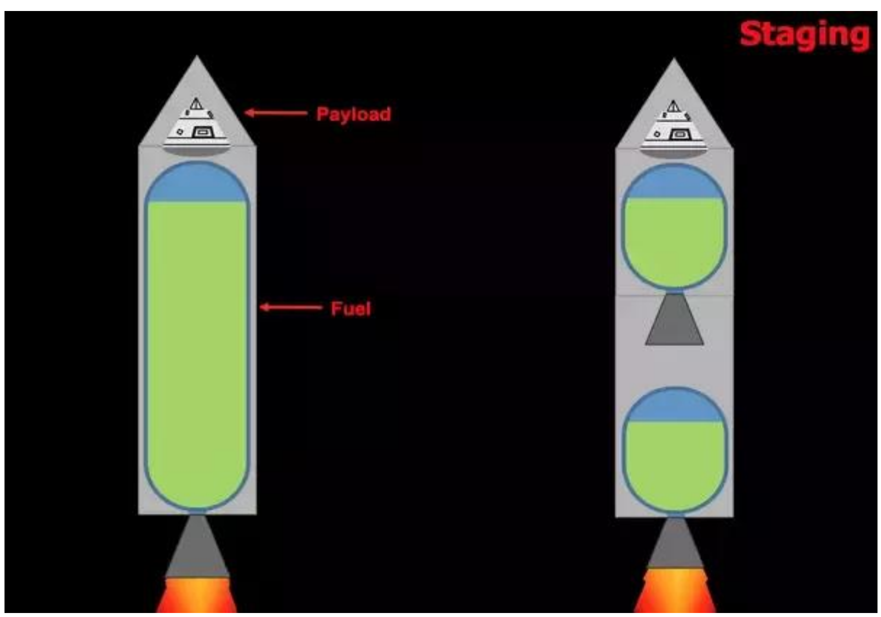

Fig 1.3 Single stage and double stage rockets at the time of ignition

The two rockets halfway through their flight. The single stage has a lot of empty fuel tank mass that we are having to carry with us. The multi-stage has dropped its empty fuel tank and become a smaller, leaner, more effective rocket.

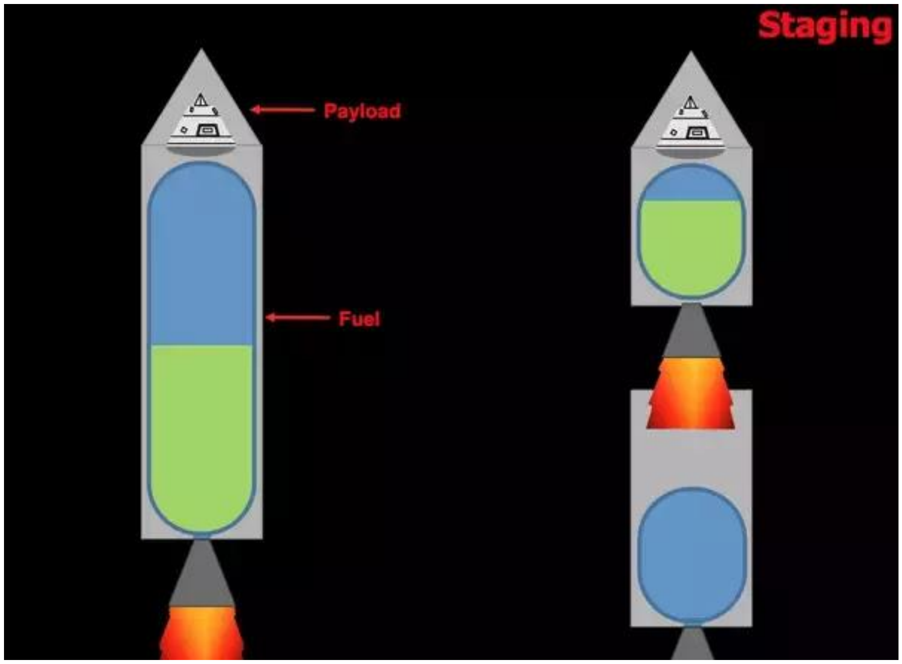

Fig 1.4 Single stage and double stage rockets at the time of completion of first stage propellant

The hardest a rocket has to work is immediately at launch as it is trying to lift the most mass (all that unburned fuel) and doing so in the greatest gravitational environment (closest to Earth) and in the thickest atmosphere. That means a bigger engine is needed at launch than higher up. While the single stage has to keep using that oversized engine, the multi-stage can drop that big heavy engine and start using one designed for vacuum only. No, solid boosters are not essential. Solid boosters are very effective, but many rockets do not use them. For example, the most powerful rocket ever used was the Saturn V that took us to the moon. It did not use solid boosters.

1.11 History Of Staging Of Rockets:

In the 14th century,the oldest known multistage rocket; this was the 'fire-dragon issuing from the water' used mostly by the Chinese navy. It was a two-stage rocket that had booster rockets that would eventually burn out, yet before they did they automatically ignited a number of smaller rocket arrows that were shot out of the front end of the missile, which was shaped like a dragon's head with an open mouth.This multi-stage rocket may be considered the ancestor to the modern The British scientist and historian Joseph Needham points out that the written material.

Another example of an early multistaged rocket is the *Juhwa* of Korean development. It was proposed by medieval Korean engineer, scientist and inventor Choe Museon and developed by the Firearms Bureau during the 14th century.[1] The rocket had the length of 15 cm and 13 cm; the diameter was 2.2 cm. It was attached to an arrow 110 cm long; experimental records show that the first results were around 200m in range.[1] There are records that show Korea kept developing this technology until it came to produce the Singijeon, or 'magical machine arrows' in the 16th century. The earliest experiments with multistage rockets in Europe were made in 1551 by Austrian Conrad Haas(1509–1576),The first high-speed multistage rockets were the RTV-G-4 Bumper rockets tested at the White Sands Proving Ground and later at Cape Canaveral from 1948 to 1950. These consisted of a V-2 rocket and a WAC Corporal sounding rocket. The greatest altitude ever reached was 393 km, attained on February 24, 1949, at White Sands.

In 1947, the Soviet rocket engineer and scientist Mikhail Tikhonravov developed a theory of parallel stages, which he called "packet rockets". In his scheme, three parallel stages were fired from liftoff, but all three engines were fuelled from the outer two stages, until they are empty and could be ejected. This is more efficient than sequential staging, because the second-stage engine is never just dead weight. In 1951, Soviet engineer and scientist Dmitry Okhotsimsky carried out a pioneering engineering study of general sequential and parallel staging, with and without the pumping of fuel between stages. The design of the R-7 Semyorka emerged from that study. The trio of rocket engines used in the first stage of the American Atlas I and Atlas II launch vehicles, arranged in a "row", used parallel staging in a similar way: the outer pair of engines existed as a jettisonable pair which would, after they shut down, drop away with the lowermost outer "skirt" structure,leaving the central "sustainers" engine to complete the first stage's engine burn towards apogee or orbit.

1.12 Satellite Orbits

Satellites travel around the Earth along predetermined repetitive paths called orbits. Fig 1.5 represents an elliptical orbit with one focus at the Earth’s center.

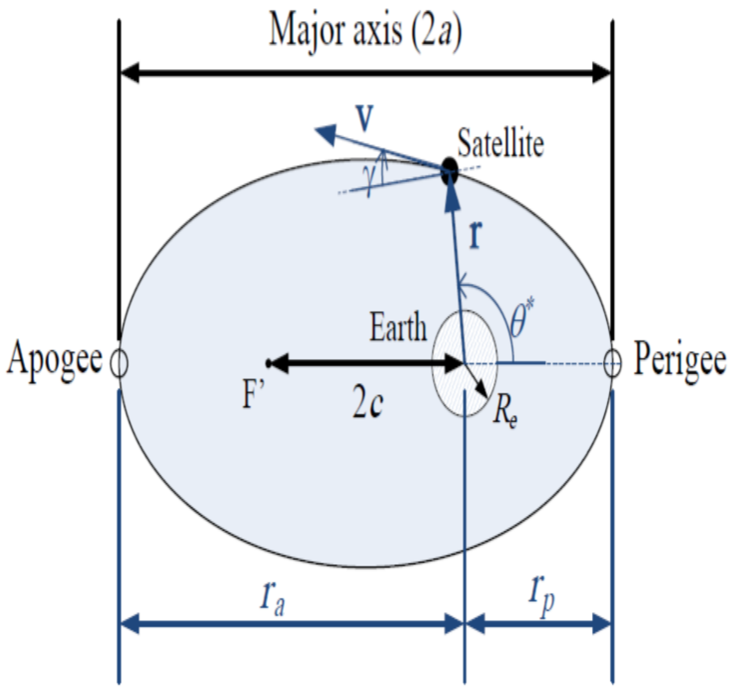

Fig 1.5 Elliptical orbit

The apogee is the point on the orbit that is farthest from the Earth's center; whereas theperigee is the point closest to the Earth's center. Distance from the apogee to the Earth's center is called the apogee radius(r_a) and the distance from the perigee to the Earth's center is called the perigee radius (r_p).The apogee altitude (h_a) and the perigee altitude (h_p) are the heights above the Earth's surface and expressed as

$$h = r_a - R_e \quad (1)$$

$$h_p = r_p - R_e \quad (2)$$

In Eqs. (1) and (2),

R_e is the Earths's equatorial radius, i.e. $R_e = 6378.137$ km.

1.12.1 Orbital Elements

There are six parameters required to uniquely identify a specific orbit and they are called classical orbital elements, also known as Keplerian parameters.The two main elements, which are the semimajor axis(a) and the eccentricity (e), respectively define the size and the shape of the ellipse.

Semimajor axis (a) defines the size of the orbit and it is the half length of the major axis of the ellipse.

$$a = \frac{r_a + r_b}{2} \qquad (3)$$

Eccentricity (e) defines the shape of the orbit and it is the ratio of the distance between the two foci to the length of the major axis.

$$e = \frac{c}{a} = \frac{r_a - r_p}{r_a + r_p} \qquad (4)$$

For a circular orbit e = 0 (a = r); whereas for an elliptical orbit
0 < e < 1 (a > 0). Trajectory is parabolic when e = 1 (a = ∞) and hyperbolic when e > 1 (a < 0).

Two elements, namely the inclination (i) and the right ascension of the ascending node (Ω) define the orientation of the orbital plane in space (Fig 1.6).

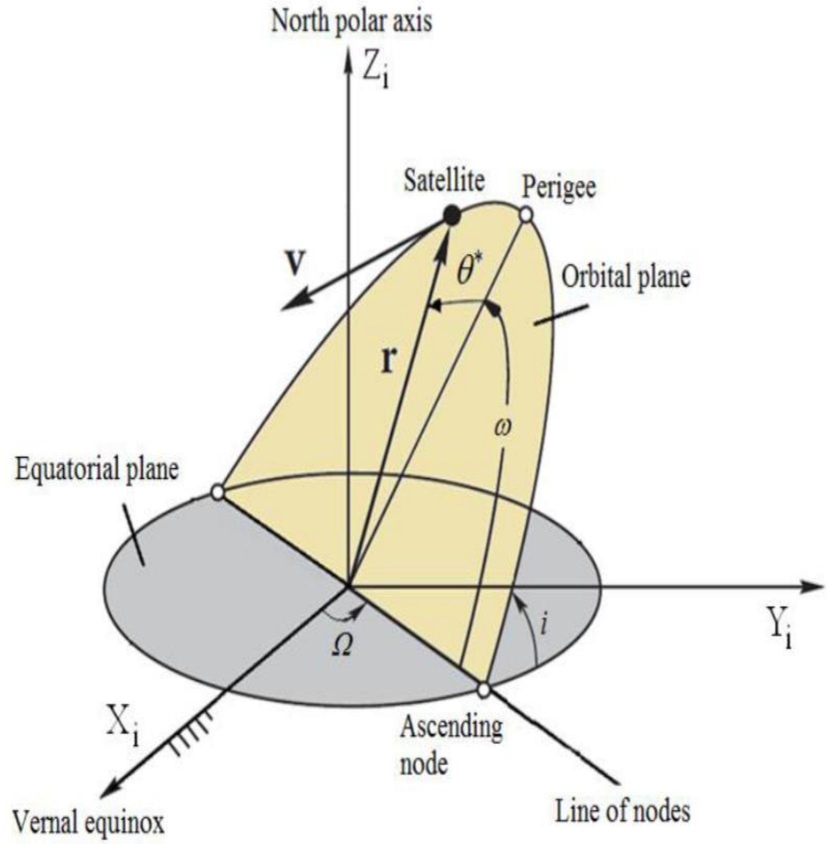

Fig 1.6 Orbital elements

Inclination (i) is the angle between the orbital plane and the equatorial plane and takes values in the range of 0-180°.It is equal to zero for equatorial orbits and 90°for polar orbits. Inclination can be determined by the following well known relation.

$$\cos i = \sin A_0 . \cos \delta_0 \qquad (5)$$

In Eq. (5),

A_0 is the inertial launch azimuth (angle measured clockwise from north),δ_0 is the geocentric latitude of the launch site.

Eq. (5) can be satisfied if and only if i ≥ δ_0. Therefore, achievable inclinations are constrained by the latitude of the launch site. It is impossible to launch directly into an inclination lower than the launch site's latitude without orbit plane transfer which requires a large amount of velocity change (ΔV). Therefore, launch sites at
or near the Equator are highly desirable, because launch into any inclination from them are possible.

Right ascension of the ascending node –RAAN (Ω) is the angle between the vernal equinox direction and the ascending node. The ascending node is the point where the orbit crosses the equatorial plane when the satellite passes from the southern hemisphere to the northern hemisphere, and the vernal equinox is the vector pointing the fixed stars in the constellation of Aries (Figure A.2). On the first day of spring, line joining the Earth's center and the Sun's center points invernal equinox direction.

RAAN takes values in the range of 0-360° and is undefined when i = 0° or i =180°. Specified RAAN can be achieved by choosing an appropriate injection time depending upon the longitude of the injection point as Maini & Agraval (2011) emphasized. Tewari (2007) derived a useful relationship to determine RAAN utilizing the spherical trigonometry.

$$\Omega = \lambda - \arcsin\left(\frac{tan\delta}{\tan i}\right) \qquad (6)$$

In Eq. (6),
λ is the geodetic longitude,
δ is the geocentric latitude.

Argument of perigee (ω) defines the orientation of the ellipse (in which directionit is flattened compared to a circle) in the orbital plane, as an angle measured fromthe ascending node to the perigee. Argument of perigee takes values in the range of 0-360° and is undefined when $i = 0°$ or $i = 180°$ or $e = 0$. As Maini & Agraval (2011) noted that ω can directly be calculated from the following relation when the injection point is the same as the perigee point.

True anomaly (θ*) defines the position of the satellite along the orbit at a specific time and it is the angle between the perigee and the satellite location. True anomaly takes values in the range of 0-360° and is undefined when $e = 0$. At perigee and apogee points, $\theta^* = 0°$ and $\theta^* = 180°$, respectively.

A real orbit (and its elements) changes over time due to gravitational perturbations by other objects and the effects of the relativity. A Keplerian orbit is an idealized mathematical approximation and true anomaly is assumed as the only orbital element that changes with time. As mentioned above, some of the orbital elements become undefined for certain special cases (Table 1).

Table 1 Special cases of orbits

Type of orbit	**Undefined parameter**
circular ($e = 0$)	ω & θ*: undefined
equatorial ($i = 0$)	Ω & ω: undefined
circular and equatorial ($e = 0$ & $i = 0$)	Ω, ω & θ*: undefined

CHAPTER 2

2. LITERATURE REVIEW

EzgiCivek-Coskunet.al[1] The staging optimization problem for multistage rockets which carry payloads from theEarth's surface into the Earth orbits. In the early design phases, requirements are not so strict, there are many unknowns and problem arises as to what is the optimum staging to achieve thegiven mission. Therefore, designers need simplified tools providing a quick insight on the vehicle performance with minimum basic vehicle data. For this purpose, a Matlab® basedcomputer program has been written to determine staging parameters (number of stages, mass distribution between stages, and the propellant and structural masses for each individualstage) which minimize the gross lift-off mass of the launch vehicle for a specific mission.In this study, staging optimization problem has been formulated based on Delta-V equations and solved by method of Lagrange Multipliers. The problem has been stated in a generalform to handle launch vehicles having arbitrary number of stages and with various configurations involving serial, parallel and clustered stages; and with different structural ratios and propellant exhaust velocities in each stage. Staging optimization program developed in this study has been verified for different missions using available data of existing launch vehicles. Thus, a quick and effective tool to find optimal vehicle configurations in the conceptual design phase of a generic multistage launch vehicle has been achieved.

H.H.Hallet.al[2] The optimum weight distribution for multistage rockets having different specific impulses and structural factors in each stage is derived. Minimization of gross weight for a given required burnout velocity and payload is the criterion of optimization used. A method is suggested for including in first approximation the effects of gravity, drag and turn.

David N. Burgheset.al[3] In this paper the fundamental characteristics of rocket staging are described.The equation ofmotion of a rocket is derived, and it is demonstrated that single stage rockets are not able to launch earth satellites successfully. Two-stage rockets are analysed and the optimum choice of the rocket stage masses is found for maximum final speed. Multistage rockets are then considered and again individual stage masses are found for maximum final speed with constant total mass. This maximum final speed is evaluated for varying number of stages, and it is shownthat the optimum choice for n is 2 or 3 for most earth satellite launching operations.

F J Malinaet.al[4] The necessary velocity to escape from the earth is re-examined in the light of recently released information on wartime rockets. The fundamental equations of motion of a rocket in outward radial flight are derived and the influence of each of the following design parameters is examined: c, the effective jet velocity; the ratio of propellant mass to initial mass; tVt the time of powered flight; M, the ratio of initial mass to maximum cross-sectional area, the drag coefficient based on the same cross-sectional area.

***T.N.Srivastava*et.al[5]** Optimum staging programme for step rockets of arbitrary number of stages having different specific impulses and mass fractions with stages is derived, the optimization criterion being minimum take-off weight for a desired burntout velocity at an assigned altitude. Variation of thrust attitude angle from stage to stage and effects of gravity factor are taken into account. Analysis is performed for a degenerate problem obtained by relaxing the altitude constraint and it has been shown that problems of Weisbord, Subotowicz, Hall &Zambelli and Malina& Summerfield are the particular cases of the degenerate problem.

M. Subotowiczet.al[6] This paper gives the derivation of equations for determining the optimum weight distribution for two-stage tandem for the case of different construction parameters and propellant specific impulses in each stage. In the paper, the results were generalized for the case of then stage tandems, arbitrary number of stages, with different construction parameters and propellant specific impulses in each stage. All principal equations for the optimized two-stage rocket are easy to obtain from our equations for an optimized n-step rocket. The principal results presented in this paper were worked out in 1955 a n d werepresented by the author in May 1957 at the Conference on Rocket Techniques and Astronautics in Warsaw, Poland. The criterion of optimization used here is the minimization of gross weight for a given required burn out velocity and pay load.

V. B. Tawakleyet.al[7] The effect of gravity on the optimum distribution of total required mass among the various stagesof a multiple stage rocket arranged series has been considered by making the payload ratiominimum so as to obtain a specified mission velocity at the end of powered flight. The special casewhen the physical parameters for all the step rockets are each equal, has been discussed in detail.It has also been shown that if the mission requirement is to achieve a given all burnt height, theneven at the expense of more total initial mass no more total a given all burnt velocity. Finally it isproved that in order to achieve a given all burnt velocity by arranging the stages in parallel resultsin an increase in the total initial mass compared to the case when they are arranged in series andthe magnitude of this increase depends upon the number of stages.

C.N. Adkinset.al[8] In this paper addressing the problem of minimizing vehicle weight for specified payload and velocity at burnout generally either 1) permit a different specific impulse / for eachstage,1-2 but no provision for acceleration constraints, gravity, turning, or drag; or 2) provide for acceleration constraints and gravity,3 but not for turning, different 7's or drag. Reference 4, which maximizes payload total energy, does include acceleration constraints and the average effects of turning and / variation, but does not give an explicit formulation for determining these averages. In this Note, these formulations are included, as well as expressions for the average effects of drag for each stage. In addition, the angle between thrust and velocity vectors a will be considered in the aerodynamic sense (angle of attack) for those stages which include drag and as the angle for thrust vector control of the vehicle trajectory for those stages which neglect drag (exoatmospheric). The idea behind staging is to improve performance by reducing the vehicle's mass on the way to orbit. Once the propellant of a stage is consumed, the empty stage which is no longer useful and only adds weight to the vehicle is discarded and the next stage is ignited. This stage then accelerates the rest of the vehicle much faster. As a result, less propellant is required to reach the desired orbit.

SaqlainAkhtar and He Linshuet.al[9] Conceptual design refers to systems studies conducted early in the design process and intended to reveal trends and allow relative comparisons among alternatives. Such conceptual design studies provide quantitative data that can be used by decision makers while the design is still flexible and before the greatest share of life cycle costs are committed.The major objective of this paper is to find the computational effectiveness and efficiency of the hybrid method first using GA for global space exploration then incorporating gradient based methods to fine-tune local solutions. This combination of methods in parallel has the promise of being superior to either method alone.

CHAPTER 3

3. THEORETICAL CALCULATIONS

3.1 Basic Rocket Equation for Velocity Increment in a Time Interval:

After time ‘t’ from starting during a small time period of Δt (as lim $\Delta t \to 0$)

m,$v^{(\Delta t)}$,(m-Δm)($\vartheta + \Delta v$)

For a launch vehicle:

Δm = change in mass in (Δt) time, $\Delta\vec{v}$ = change of velocity in (Δt) time,

From impulse–momentum principle

$$Lt_{\Delta t\to 0}\int_0^{\Delta t}\overrightarrow{F_1}\,\mathrm{dt} = (\text{m-}\Delta m)(\vec{v}+\overrightarrow{\Delta v}) - (\text{m-}\Delta m)\vec{v}$$

$\overrightarrow{F_1}$ is constant as $\Delta t \to 0$

$\overrightarrow{F}$,(Δt) = m$\overrightarrow{\Delta v}$

Similarly for jet gases

$$Lt_{\Delta t\to 0}\int_0^{\Delta t}\overrightarrow{F_2}\,\mathrm{dt} = \Delta m(-V_j) - \Delta m\vec{v}$$

$\overrightarrow{F_2}$,(Δt) = $\Delta m(C_j\,)$

Since $\lfloor\overrightarrow{F_1}\rfloor = -\lfloor\overrightarrow{F_2}\rfloor$

$$\frac{M\overrightarrow{\Delta v}}{(\Delta t)} = \frac{-\Delta m(\overrightarrow{cj})}{\Delta t}$$

$$\overrightarrow{\Delta v} = \frac{-\Delta m}{m}\overrightarrow{cj}$$

cj = velocity of exhaust gas with respect to nozzle

For over a period of time't' we should integrate them as,

$$Lt_{\Delta t \to 0} \frac{\Delta v}{\Delta m} = \frac{d\vec{v}}{dm}$$

$$\int_{vi}^{vf} d\vec{v} \; = - \left[-\int_{mi}^{mf} \frac{dm}{m} \vec{cj} \right]$$

$$(\overrightarrow{\Delta v}) \;\; = \; - \vec{cj} \ln\left[\frac{mi}{mf}\right]$$

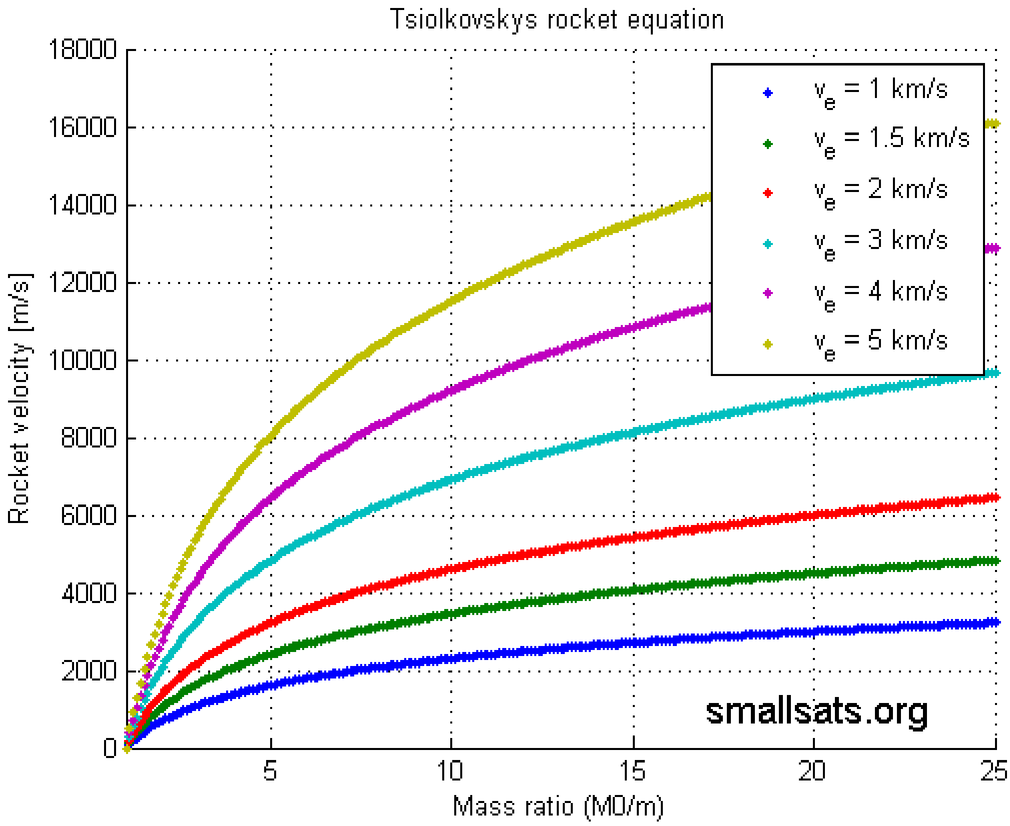

Fig 3.1 Plot shows variation of velocity of rocket with mass ratio for various ejection

gases velocites.

$\{\overrightarrow{\Delta v}$ = over period of time t $\}$ $\overrightarrow{\Delta v} = -\vec{cj} \ln[\wedge]$

$\{ \wedge = \frac{mi}{mf}$ = mass ratio $\}$Rocket equation: $[\overrightarrow{\Delta v}]_{ideal} = |\vec{cj}| \ln[\wedge]$

For multistage rockets having n stages

$$|\Delta v|_{vehicle} = \sum_{k=1}^{n} |cj_k| \ln[\Lambda_k]$$

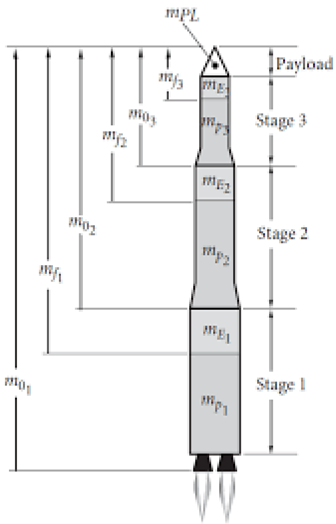

Fig 3.2 serial staging

3.2 Serial Staging

For each stage

$$Cj_k = Isp_k(g)$$

Mass ratio $= \Lambda_k = \frac{m_i,k}{m_f,k} \Rightarrow \frac{ms,k+mp,k+mpl,k}{ms,k+mpl,k} = \frac{mo,k}{ms,k+mpl,k}$

Structure ratio $\varepsilon_k \Rightarrow \frac{ms,k}{ms,k+mp,k}$

Payload ratio $= \lambda_k \Rightarrow \frac{mpl,k}{ms,k+mp,k}$

$$\Lambda_k = \frac{1+\lambda_k}{\varepsilon_k+\lambda_k}$$

3.3 Parallel Staging

For parallel staging an equivalent serial staging has determined and all parallel boosters along with the propellant of core stage which burnt along with boosters is considered as zeroth stage

$M_{bk} = mass\ of\ booster\ \ stage$

$M_{p01} = propellant\ mass\ of\ the\ core\ stage\ burn\ during\ the\ zero\ th\ stage$

M_b $= mass\ of\ all\ parallel\ boosters$

$\Lambda_o = \frac{m_{kb+m_{01}+m_{p01}}}{m_{sb} + m_{o1}}$

$\varepsilon_o = \frac{m_{sb}}{m_{kb} + m_{po1}}$

$$\lambda_o = \frac{m_{o1}}{mk_b + m_{p1o}}$$

For equivalent zeroth stage

$[I_{sp}]_{avg} = \frac{[I_{sp}]_b m_{pb} + [I_{sp}]_c m_{p1o}}{m_{pb} + m_{p1o}}$

$[c_{sp}]_o = [I_{sp}]_{avg} \times g$

In general range of specific impulse $250 < I_{SP} < 475$

And the range of structural ratio $0.1 < \mathcal{E}_k < 0.2$

Orbital velocity equation for an elliptical path by conservation of energy

KƐ+PƐ = constant

$\frac{1}{2}mv^2 - \frac{GMm}{r}$ = constant ——— (i)

3.4 Orbital Velocity at a Radial Distance from Focal Point

By keplers law due to the conservation of angular momentum principle in equal time interval object covers equal area with the focal point.

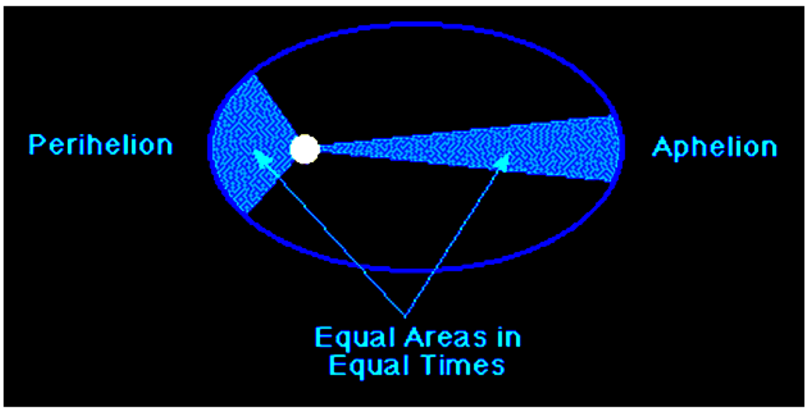

Fig 3.3 Keplers second law

$$A_1 = A_2$$

$$\frac{1}{2}(r_1\theta_1)r_1 = \frac{1}{2}(r_2\theta_2)r_2$$

$$\frac{1}{2}\frac{(r_1\theta_1)}{\Delta t}r_1 = \frac{1}{2}\frac{(r_2\theta_2)}{\Delta t}r_2$$ [if time period lt $\Delta t \rightarrow 0$]

$$v_1 r_1 = v_2 r_2 \text{ ------ } \quad \text{(ii)}$$

$$\frac{\theta_1}{\Delta t} = \square_1 \,,\ \frac{\theta_2}{\Delta t} = \square_2$$ as Δt is very small

From eq (i)

$$\frac{1}{2}\text{m}v_1^2 - \frac{GMm}{r_1} = \frac{1}{2}\text{ m}v_1^2\left[\frac{r_1}{r_2}\right] - \frac{GMm}{r_2}$$

$$\frac{1}{2}\text{ m}v_1^2\left[\frac{r_2^2 - r_1^2}{r_2^2}\right] = \text{GMm}\left[\frac{r_{2-}r_1}{r_1 r_2}\right]$$

$$\frac{1}{2}m_1 v_1^2\text{ [2a]} = \text{GMm}\left[\frac{r_2}{r_1}\right]$$

$$v_1 = \sqrt{\frac{GM}{a}\left(\frac{r_2}{r_1}\right)}$$

Similarly for any radial location point(P) at radius(r) from focal point with velocity(v)

$$\frac{1}{2}\text{m}v_1^2 - \frac{GMm}{r_1} = \frac{1}{2}\text{ m}v_1^2 - \frac{GMm}{r}$$

$$\frac{1}{2}\text{m}\left[\frac{GM}{a}\frac{r_2}{r_1}\right] - \frac{GMm}{r_1} = \frac{1}{2}\text{ m}v^2 - \frac{GMm}{r}$$

$$\text{GMm}\left[\frac{1}{r} - \frac{1}{2a}\right] = \frac{1}{2}\text{ m}v^2$$

$$v_{orbital} = \sqrt{GM\left[\frac{2}{r} - \frac{1}{a}\right]}$$

3.5 Velocity Gain due to Earth rotation at launch location

Launch azimuth

Since, $A_o = \sin^{-1}[\frac{\cos i}{\cos \delta_o}]$

Linear velocity due to rotation of the earth at launch location

$V_{r,\emptyset} = \omega_e r_o \cos \delta_o$

The required velocity

$\Delta V_{req} = \sqrt{\left[v_{orbital} \sin A_o - V_{(r,\emptyset)}\right]^2 + [V_{orbit} cosA_o]^2}$

Velocity gain due earth rotation

$\Delta V_{gain} = \Delta V_{orbit} - \Delta V_{req}$

3.6 Trajectory Losses

The actual velocity a rocket can generate can be approximated by subtracting the appropriate trajectory losses from the ideal velocity as follows

$$\Delta v = g_o I_{sp} m_o \ln \frac{W_O}{W_f} - \int_{t_o}^{t_f} g sin\gamma \, dt - \int_{t_o}^{t_f} \frac{D}{m} dt - \int_{t_o}^{t_f} \frac{F}{m}(1 - cos \propto) dt$$

The first term in this equation is the ideal velocity that we have already calculated for the two cases. The other three terms represent the gravity loss, dragloss, and steering loss,respectively.in the real world situations, all three of these losses are usually evaluated by numerical integration.

Gravity losses arise because part of the rockets energy is wasted in holding it aloft and in pushing it against the relentless pull of earth's gravity. The gravity loss equation

$$\int_{t_o}^{t_f} g sin\gamma \, dt$$

Represents the numerical integral from the ignition point to the burn out point where g is the local gravitational acceleration, and γ is the flight path angle (instantaneous angle between the velocity vector and the local horizontal).

The drag loss is caused by the friction between the rocket and the ambient air. It can be expressed as

$$\int_{t_o}^{t_f} \frac{D}{m} dt$$

Where both the drag force, D, and the mass of the rocket, m, are continuously changing. The instantaneous drag force, for example, is a strong function of the rockets current velocity and the local density of the atmosphere.

The steering loss arises because the instantaneous thrust vector is not always parallel to the current velocity vector. This small mismatch is necessary otherwise, we could not steer the rocket along an optimal trajectory as it flies into space. The steering loss can be evaluated from the following expression:

$$\int_{t_f}^{t_f} \frac{F}{m}(1 - cos \propto)dt$$

Where F is the current thrust of the rocket is the current mass, and α is the steering angle, the angle between the thrust vector and the current velocity vector.

For a typical rocket of modern design flying into a low altitude earth orbit, these various losses amount to about 5000 feet per second. The velocity required to reach a low altitude orbit is around 25000 feet per second.

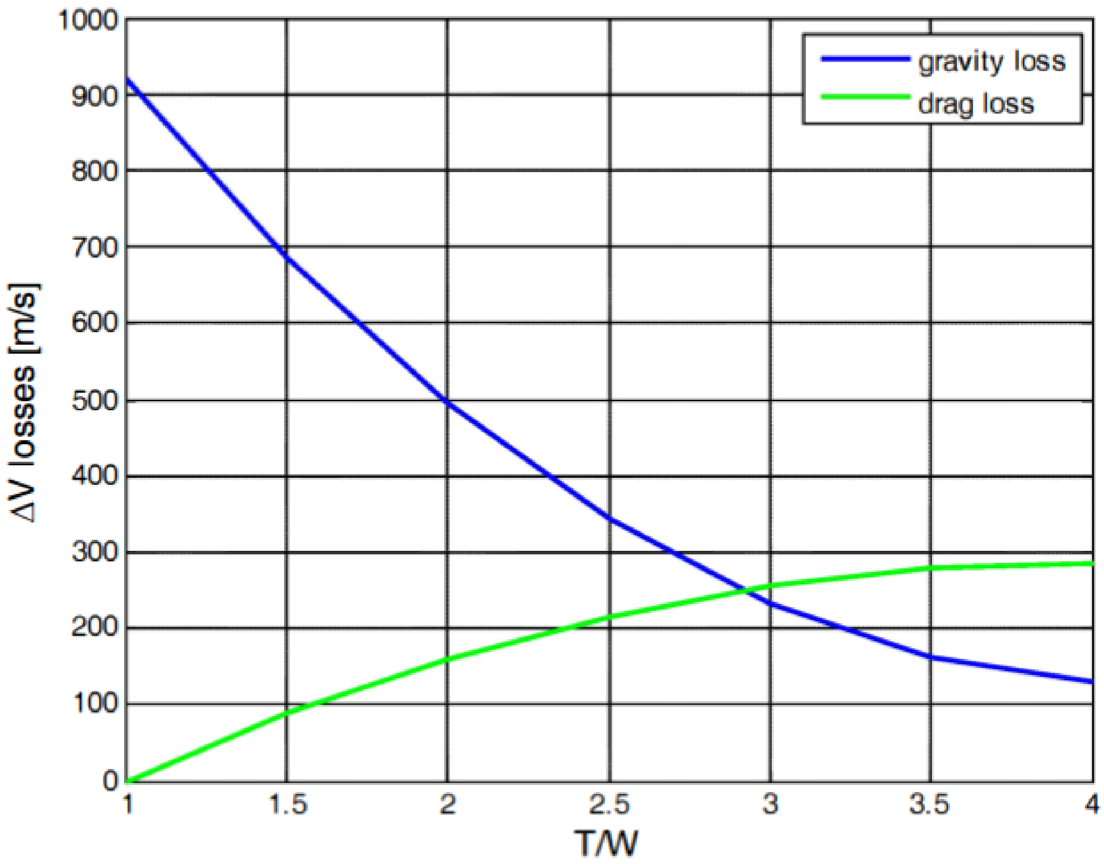

Fig 3.4 a plot showing gravity and drag loses of a vertical take off launch vehicle with respect to its initial thrust to weight ratio

Velocity loss due to gravity [m/s]

V_g = 81.006*TW^2 – 667.62*TW + 1505.4;

Velocity loss due to aerodynamic drag [m/s]

V_d = -32.962*TW^2 + 258.86*TW – 226.57;

From reference of Tewari, he proposed to add a total of 1.5km/s margin for the possible velocity losses and gains. For a rocket launched to a lower earth oribit and 2km/s applied to a rocket launched to GTO orbit

3.7 ∆V Mission Formation

$$\Delta v_{mission} = \Delta v_{orbital} + \Delta v_{gravity} + \Delta v_{drag} + \Delta v_{propolsive} - \Delta v_{gain} + \Delta v_{performance\ margin}$$

(Δv_g &Δv_{drag} are known by thrust to weight ratio,Δv_{gain} is due to earth rotation depends on leaving point of launch vehicle.)

3.8 Formation of function to be optimized

Total mass m_o= m_o,1 = $\left[\sum_{k=1}^{n}\left(m_s, k + m_p, k\right)\right]$ +mpl

$$\frac{m_o}{mpl} = \frac{m_o,1}{mpl,2} \times \frac{mpl,1}{mpl,2} \times \frac{mpl,2}{mpl,3} \times \cdots\cdots\times \frac{mpl_{n-1}}{mpl}$$

$$\frac{mo}{mpl} = \frac{mo,1}{mpl,1} \times \frac{mo,2}{mpl,2} \times \cdots\cdots\times \frac{mo,n}{mpl,n}$$

(GLOM) $m_o = \left[\prod_{k=1}^{n} \frac{m_o,k}{mpl,k}\right]$mpl

$$m_o = \left[\prod_{k=1}^{n} \frac{(1-\varepsilon_k)\wedge_k}{\left(1-\varepsilon_{k\wedge_k}\right)}\right]\text{mpl}$$

Then we have to minimize m_o

m_o = f$(n, \varepsilon_k, \wedge_k, mpl)$

Input => n,ε_k, mpl

Then m_o = f ($\wedge_k$)

Since m_o>1 then for m_o(min) then ln(m_o) is minimum launch vehicle must provide

$\Delta v_{vehicle} = \sum_{k=1}^{n} c_j, k \ln(\wedge k)$

By using LaGrange's multipliers we find minimum (GLOW) M_O .

f^{*}=ln$[f]$ + P g

$$f^{*} = \ln\left[\prod_{k=1}^{n}\left[\frac{(1-\varepsilon_k)\Delta_k}{\left(1-\varepsilon_{k\Delta_k}\right)}\right]mpl\right] + P\left[\sum_{k=1}^{n} c_j, k \ln \Delta k\right] - \Delta v_{vehicle}]$$

P = Lagrange multiplier

$$f^{*}=\sum_{k=1}^{n} \ln\left[\frac{(1-\varepsilon_k)\wedge k}{\left(1-\varepsilon_{k\wedge_k}\right)}\right] + \ln[mpl] + P\ [\sum_{k=1}^{n} cj, k \ln[\wedge k] - \Delta v_{mission}]$$

$$f^{*}=\sum_{k=1}^{n}\left[\ln\left[\frac{(1-\varepsilon_k)\wedge_k}{1-\varepsilon_{k\wedge_k}}\right] + Pcj_k \ln[\wedge k]\right] + \ln[mpl] - P\ \Delta v_{mission}$$

For stationary point

$$\frac{\partial f^{*}}{\partial \wedge_k} = 0 \quad \frac{(1-\varepsilon_k)}{\wedge_k(1-\varepsilon_k)} + \frac{\varepsilon_k}{\left(1-\varepsilon_{k\wedge_k}\right)} + P\,\frac{cj_k}{\wedge k} = 0$$

$$[1 + Pcj_k] = \frac{\varepsilon_k}{\left[\varepsilon_k - \frac{1}{\wedge k}\right]}$$

$$\wedge \mathrm{k} = \frac{1+Pcj_k}{P\varepsilon_k cj_k}$$

Since $\Delta v_{mission} = \sum_{k=1}^{n} cj_k \ln[\wedge k]$

$$\Delta v = \sum_{k=1}^{n} cj_k \ln\left[\frac{1+Pcj_k}{P\varepsilon_k cj_k}\right]$$

Let $Y = \Delta v - \sum_{k=1}^{n} cj_k \ln\left[\frac{1+Pcj_k}{P\varepsilon_k cj_k}\right] = 0$ (constrained function)

By newton raphson (technique) method y= f(P)

P will be the root of equation function y

3.9 For initial guess of root

For $\ln\left[\frac{(1-\varepsilon_k)\wedge k}{(1-\varepsilon_k \wedge_k)}\right]$

If $\varepsilon_k < 1$

$\varepsilon_k \wedge_k < 1$

$1 < \wedge_k < \frac{1}{\varepsilon_k}$ (condition – 1)

$1 < \frac{1+Pcj_k}{P\,cj_k\varepsilon_k} < \frac{1}{\varepsilon_k}$(since λ may or may not be positive)

- $\Rightarrow \frac{1}{Pcj_k} + \frac{1}{\varepsilon_k} < \frac{1}{\varepsilon_k}$
- $\Rightarrow \frac{1}{Pcj_k} < 0$
- $\Rightarrow$ P< 0

And => $\frac{1+Pcj_k}{Pcj_{k\varepsilon_k}} > 1$

- $\Rightarrow \frac{1}{Pcj_{k\varepsilon_k}} > 1 - \frac{1}{\varepsilon_k}$
- $\Rightarrow \frac{1}{Pcj_k} > (\varepsilon_k - 1)$
- $\Rightarrow P > \frac{1}{cj_k(\varepsilon_k - 1)}$
- $\Rightarrow P < \frac{-1}{cj_k(1-\varepsilon_k)}$

By Newton raphson method

Since y = $f(P)$

Tangent equation passing through point (y_1, P)

y-y_1 = $f^1(P)(\lambda - \lambda_1)$

P_l be any trail value it meet P axis at y=0

P = $P_1 - \frac{y_1}{f^1(P)}$

P = $P_1 - \frac{f(P_1)}{f^1(P_1)}$

Then for all stages ∧k will obtain

$\wedge k = \frac{1+ Pcj_k}{Pcj_k\varepsilon_k}$

Since $\lambda k = \frac{(1-\wedge k\varepsilon k)}{(\wedge k - 1)}$

for the last stage payload is the mission payload $\lambda_n = \frac{mpl}{m_{k,n},}$

$m_{k,n} = \frac{mpl}{\lambda_n}$

$\lambda_n = \frac{m_{k,n}}{m_s n + mpl}$

From this structural mass of nth stage is obtained.

By using structural ratio we can get propellant mass of the nth stage

$$\varepsilon_n = \frac{ms_n}{ms_n + mpn}$$

From the above values payload mass for n-1 stage(i,e. initial mass of the nth stage) can be obtained by using this we can get stage mass ,structural mass and propellant mass for n-1 th stage ,

$$\lambda_{n-1} = \frac{m_o, n}{m_k, n-1}$$

$$mo_{n-1} = \frac{m_o, n}{\lambda_{n-1}}$$

Similarly In this manner we can get the initial mass of the first stage it is the optimized gross lift off mass for this mission.

CHAPTER 4

4. INTRODUCTION TO MATLAB

4.1 Introduction

Cleve molar in 1984, Mathworks inc introduced as a simulation tool which supports Graphical Programming and can be interfaced with other High Level languages.Initially it isdeveloped by a lecturer in 1970's to help students learn linearalgebra.It was later marketed and further developed under MathWorks Inc.(founded in 1984) – www.mathworks.comMatlab is a software package which can be used to perform analysisand solve mathematical and engineering problems.It has excellent programming features and graphics capability – easy to learn and flexible.Available in many operating systems – Windows, Macintosh, Unix, DOS.It has several toolboxes to solve specific problems.

4.2 What is MATLAB?

MATLAB stands for MATrix LABoratory.It is a high-performance language for technical computing, math and computation, algorithm development (optimized for DSP),data acquisition, modeling, simulation, prototyping data analysis, exploration, visualization scientific and engineering graphics application development, including graphical user interface and building.

4.3 Why to learn and use MATLAB?

Extensive built-in commands for scientific and engineering mathematics,simple and intuitive programming for more complex problems, standard and widely-used computational environment with many features, extensions, and links to other software.

4.4 MATLAB System

It includes Development Environment, MATLAB Mathematical Function Library, MATLAB Language, Graphics and MATLAB Application Program Interface (API).

Development Environment consists of MATLAB desktop;Editor and debugger for MATLAB programs ("m-files");Browsers for help, built-in and on-line documentation; Extensive demos. The MATLAB Mathematical Function Library consists of Elementary functions, like sum, sine, cosine, and complex arithmetic; More sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms; "Toolboxes" for special application areas such as Signal Processing. MATLAB Language consists of "Programming in the small" to rapidly create quick and dirty throw-away programs and "Programming in the large" to create large and complex application programs. Graphics consists of 2D and 3D plots; Editing and annotation features. MATLAB Application Program Interface (API) consists of A library that allows you to write C and Fortran programs that interact with MATLAB.

4.5 Types of operators

4.5.1 Arithmetic operators

plus	- Plus	+
uplus	- Unary plus	+
minus	- Minus	-
uminus	- Unary minus	-
mtimes	- Matrix multiply	*
times	- Array multiply	.*
mpower	- Matrix power	^
power	- Array power	.^
mldivide	- Backslash or left matrix divide	\
mrdivide	- Slash or right matrix divide	/
ldivide	-Left array divide	.\
rdivide	- Right array divide	./

4.5.2 Relational operators

eq	- Equal	=
ne	- Not equal	~=
lt	- Less than	<
gt	- Greater than	>
le	- Less than or equal	<=
ge	- Greater than or equal	>=

4.5.3 Logical operators

Short-circuit logical AND &&

Short-circuit logical OR ||

and - Element-wise logical AND &

or - Element-wise logical OR |

not - Logical NOT ~

xor - Logical EXCLUSIVE OR

any - True if any element of vector is nonzero

all - True if all elements of vector are nonzero

4.5.4 Bitwise Operators

bitand - Bit-wise AND.

bitcmp - Complement bits.

bitor - Bit-wise OR.

bitmax - Maximum floating point integer.

bitxor - Bit-wise XOR.

bitset - Set bit.

bitget - Get bit.

bitshift - Bit-wise shift

4.6 MATLAB windows

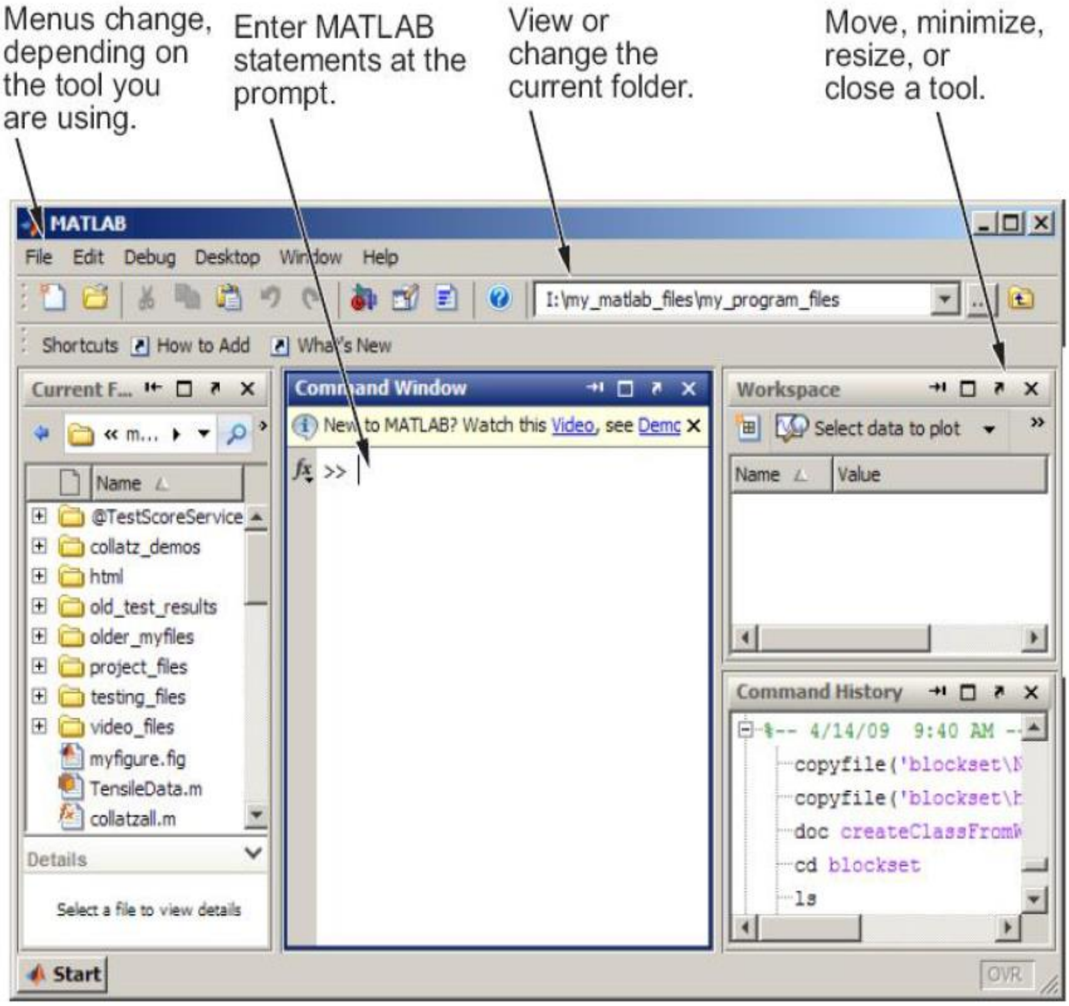

Fig 4.1 Showing various entities in MATLAB window

4.7Applications

• Aerospace

• Biometrics

• Medical

• Finance

• Control System

• Signal,Image,Audio and Video

• Neural networks,Fuzzy logic

• Animation

CHAPTER 5

5. RESULTS AND DISCUSSIONS

Vehicle 1 Ariane

Number of stages	Specific impulses	Structure ratios
N=2	281	0.0859
N=3	296	0.0976
N=4	443	0.1504
N=5	295	0.0921

Table 5.1.1 Vehicle parameters of ariane

For payload mass 300kgs

	N=2	N=3	N=4	N=5
v=8km/s	10799.7	5119.18	4962.73	
V=9km/s	19731.8	7858.58	7482.37	
V=10km/s	39659.1	12243.2	11368.7	10800.82
V=11km/s	93857.7	19422.3	17427.2	15599

Table 5.1.2 Variation of GLOM with respect to the number of stages at 300kgs Mpl for vehicle 1 at different velocities

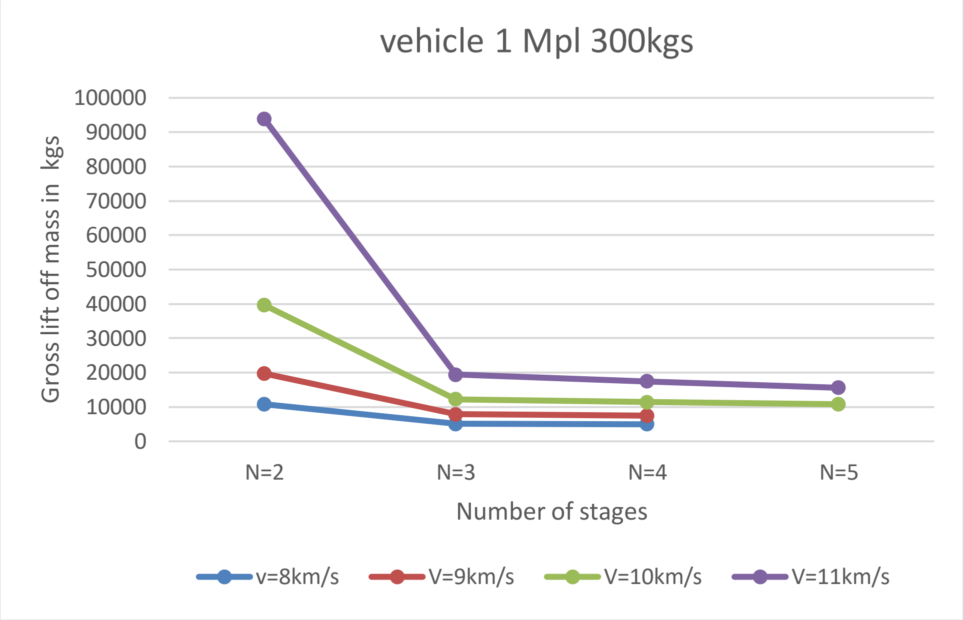

Fig:5.1.1 GLOM vs stages at 300kgs payload for vehicle 1, at various burnout velocity.

To attain velocity 8km/s,

For vehicle 1 with the above specifications, the gross lift of mass for 2nd stage is 10799.7 and for 3rd stage is 5119.18(where it decreased to 50%), for 4rth stage the gross lift off mass is 4962.3(where the decrease is less than 3%)

So 3rd stage gives the optimal solution for gross lift off mass.

To attain velocity 9km/s,

For vehicle 1 with the above specifications, the gross lift of mass for 2nd stage is 19731.8 and for 3rdstage is 7858.58 (where it decreased to 60%), for 4rth stage the gross lift off mass is 7482.37(where the decrease is less than 3%)

So 3rd stage gives the optimal solution for gross lift off mass.

To attain velocity 10km/s,

For vehicle 1 with the above specifications, the gross lift of mass for 2nd stage is 39659.1 and for 3rd stage is 12243.2(where it decreased to 50%), for 4rth stage the gross lift off mass is 11368.7 (the decrease is 8%) and for 5^{th} stage the gross lift off mass is 10800.5(the decrease is less than 5%).

So 4rth stage gives the optimal solution for gross lift off mass.

To attain velocity 11km/s,

For vehicle 1 with the above specifications, the gross lift of mass for 2^{nd} stage is 93857.7and for 3^{rd} stage is 19422.3(where it decreased to 50%), for 4rth stage the gross lift off mass is 17427.2 (the decrease is 8%) and for 5^{th} stage the gross lift off mass is 15599(the decrease is less than 5%).

So 4rth stage gives the optimal solution for gross lift off mass.

For Payload Mass=700kgs

	N=2	N=3	N=4	N=5
V=8km/s	25199.4	11944.8	11579.7	
V=9km/s	46040.9	18336.7	17458.9	
V=10km/s	92097.8	28567.5	26526.9	26400.5
V=11km/s	219001	45318.8	40663.4	38731.5

Table 5.1.3Variation of GLOM with respect to the number of stages at 700kgs Mpl for vehicle 1 at different velocities

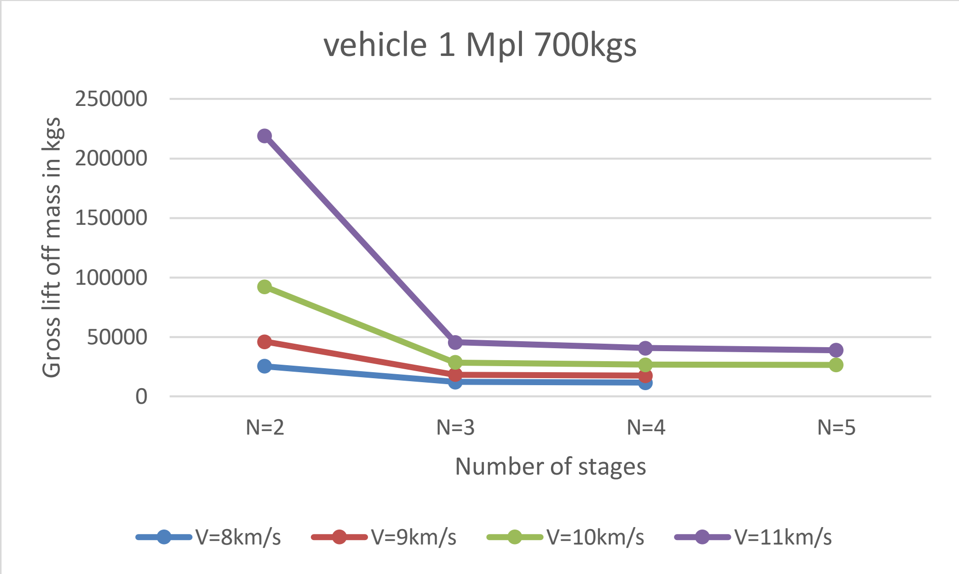

Fig:5.1.2 GLOM vs stages at 700kgs payload for vehicle 1, at various burnout velocity.

To attain velocity 8km/s

The vehicle having 3 stages gives optimal solution for gross lift of mass (as there is a decrease in 53% in GLOM from 2nd to 3rd stage)

To attain velocity 9km/s

The change is 71% from 2nd to 3rd stage, so the optimal solution for gross lift of mass is at 3rd stage

To attain velocity 10km/s

The change is 79% from 2nd to 3rd stage, and 8% from 3rd to 4rth stage and less than 2% from 4rth to 5th stage, so the optimal solution is at 4rth stage.

To attain velocity 11km/s

The change is 80% from 2nd to 3rd stage, and 12% from 3rd to 4rth stage and less than 5% from 4rth to 5th stage, so the optimal solution is at 4rth stage.

For Payload Mass=1100kgs

	N=2	N=3	N=4	N=5
V=8km/s	39599.1	18770.3	18196.7	
V=9km/s	72349	28814.8	27435.4	
V=10km/s	145417	44891.2	41685.2	40266.5
V=11km/s	344145	71215.2	63899.7	60863

Table 5.1.4Variation of GLOM with respect to the number of stages at 1100kgs Mpl for vehicle 1 at different velocities

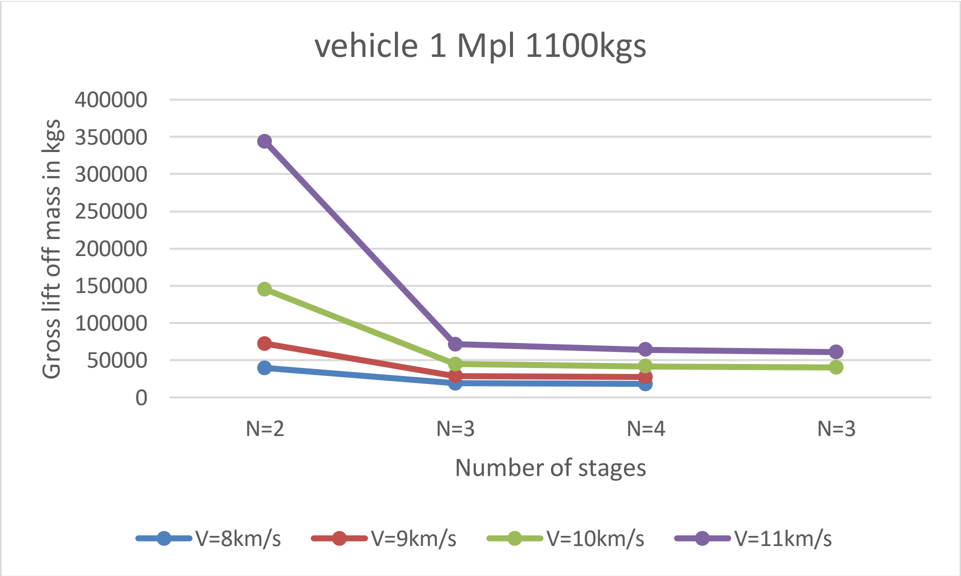

Fig:5.1.3 GLOM vs stages at 1100kgs payload for vehicle 1, at various burnout velocity.

To attain velocity 8km/s

The vehicle having 3 stages gives optimal solution for gross lift of mass (as there is a decrease in 53% in GLOM from 2nd to 3rd stage)

To attain velocity 9km/s

The vehicle having 3 stages gives optimal solution for gross lift of mass (as there is a decrease in 61% in GLOM from 2nd to 3rd stage)

To attain velocity 10km/s

The change is 70% from 2nd to 3rd stage, and 7.2% from 3rd to 4rth stage and less than 4% from 4rth to 5th stage, so the optimal solution is at 4rth stage.

To attain velocity 11km/s

The change is 80% from 2nd to 3rd stage, and 11% from 3rd to 4rth stage and less than 5% from 4rth to 5th stage, so the optimal solution is at 4rth stage.

CHAPTER 6

6. CONCLUSION

For the optimization of gross lift mass of space launch vehicles, a mat lab script written so that the optimal vehicle configuration in conceptual design phase is obtained and the variation for gross lift mass with other parameters is plotted.

For the requirement that is during a predesigned phase of a space launch vehicle which is carrying a certain amount of payload from one particular position on earth surface to a desired orbital injection position, a launch vehicle is designed such that the gross lift of mass is optimized.

We also took into account about the number of stages that are to be kept in order to get optimized solution. As lift of mass is minimized cost also minimized automatically. we used Lagrange multiplier and Newton rap son method for getting the optimization solution.npt only optimization but also the mass is distributed in each stage such that we got the optimal lift of mass and how the lift of mass varies with velocity ,payload mass and number of stages and by this we got the optimum solution.

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[11] https://en.wikibooks.org/wiki/MATLAB_Programming

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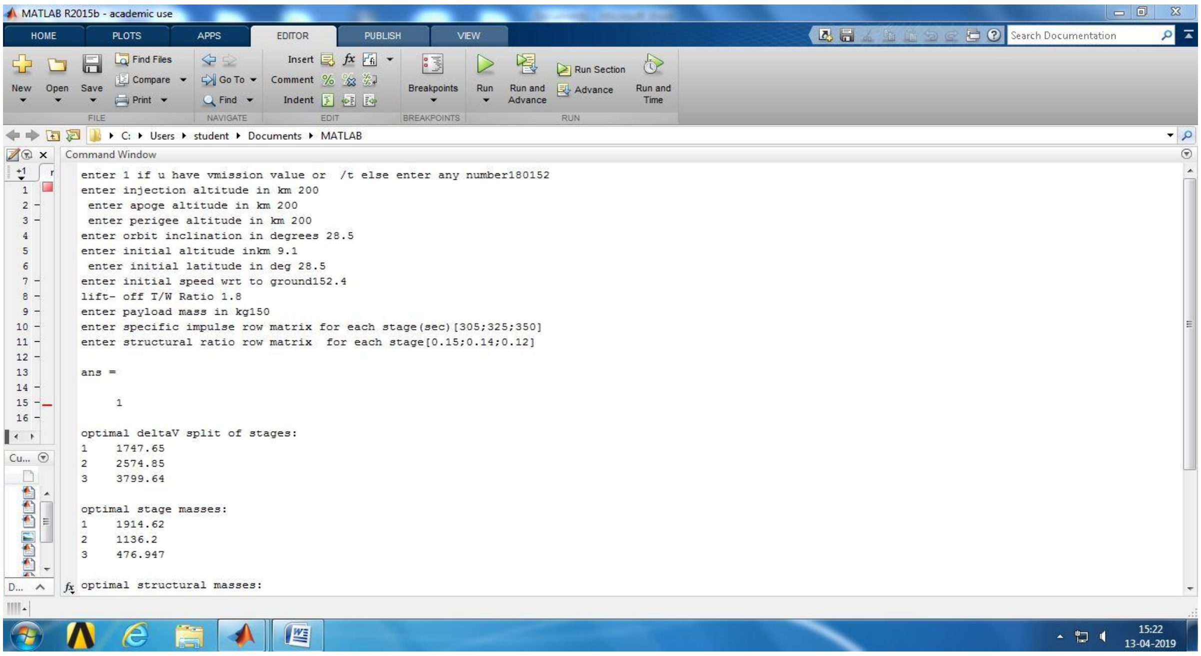
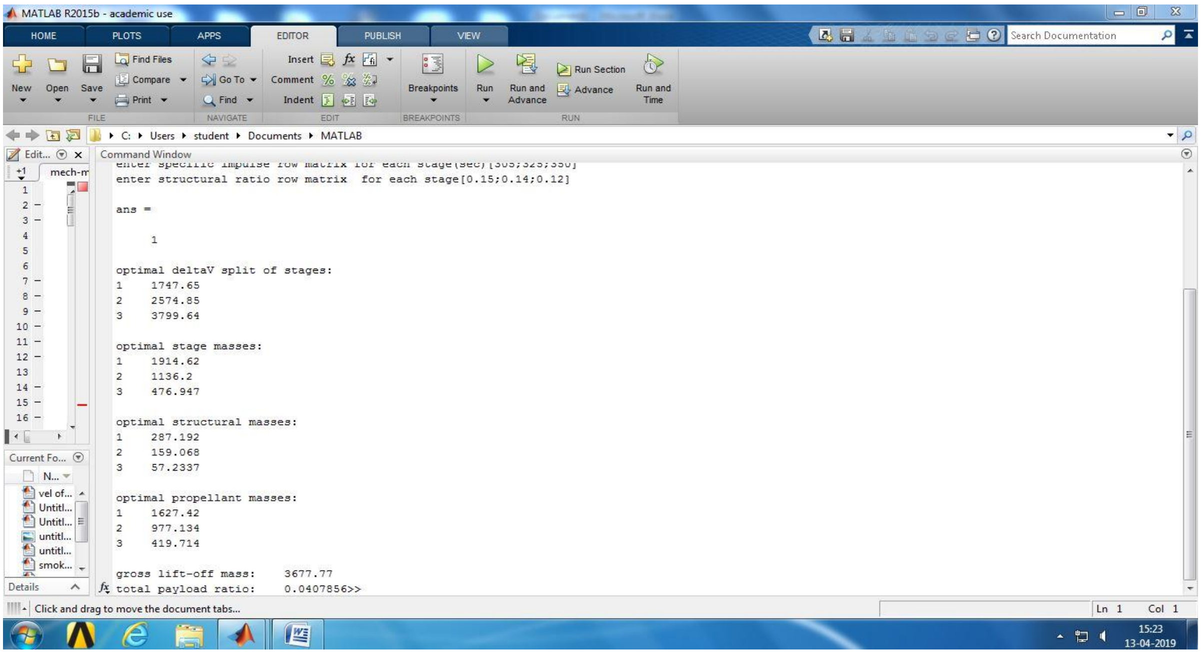
APPENDIX

```
clc
clear all
% USER INPUTS
%
========================================================================
% Mission definition
 S = input('enter 1 if u have vmission value or  /t else enter any number');
 if S ~= 1
h_inj = input('enter injection altitude in km ') ; % injection altitude (km)
h_a = input(' enter apoge altitude in km ')  ; % [km] apogee altitude
h_p =  input(' enter perigee altitude in km '); % [km] perigee altitude
i = input('enter orbit inclination in degrees ')  ; %inclination
% Launch conditions
h_0 =  input('enter initial altitude inkm '); % [km] initial altitude
phi_0 =   input(' enter initial latitude in deg '); % initial latitude
V_0 =  input('enter initial speed wrt to ground'); % [m/s] initial speed wrt ground
% Stage configuration
TW =  input('lift- off T/W Ratio ') ; % [-] lift-off thrust-to-weight ratio
% CONSTANTS
%
========================================================================
GM = 398600.4418; % [km^3/s^2] Earth's gravitational parameter
R = 6378.137; % [km] Earth's mean radius

Omega = 7.292115e-5; % [rad/s] Earth's angular velocity
d2r = pi/180; % unit conversion from degree to radian
% CALCULATIONS
%
========================================================================
% Target orbit paramete
r_inj = R + h_inj; % [km] injection radius
r_a = R + h_a; % [km] apogee radius
r_p = R + h_p; % [km] perigee radius
a = (r_a + r_p)/2; % [km] semimajor axis
% Launch parameters
r_0 = R + h_0; % [km] initial radius (@launch)
A = asin(cos(i*d2r)/cos(phi_0*d2r)); % [rad] launch azimuth
V_phi = 1e3*Omega*r_0*cos(phi_0*d2r); % [m/s] Earth's speed wrt latitude
% Vehicle configuration
% DeltaV calculations
% ----------------------------------------------------------------------
% velocity required to keep payload in a specified orbit [m/s]
```

```
V_orbit = 1e3*sqrt(GM*(2/r_inj - 1/a));
% velocity gain due to Earth's rotation [m/s]
V_rot = V_orbit - sqrt((V_orbit*sin(A) - V_phi)^2 + (V_orbit*cos(A))^2);
% velocity loss due to gravity [m/s]
V_g = 81.006*TW^2 - 667.62*TW + 1505.4;
% velocity loss due to aerodynamic drag [m/s]
V_d = -32.692*TW^2 + 258.86*TW - 226.57;
% velocity loss due to steering and pressure change [m/s]
V_p = 100;
% initial absolute velocity [m/s]
V_i = V_rot + V_0;
% margin for unexpected disturbances and inaccuracies [m/s]
V_m = 100;
% mission deltaV [m/s]
Vmission = V_orbit + V_g + V_d + V_p - V_i + V_m;
% STAGING OPTIMIZATION (minimizing gross lift-off weight, m0 while
% satisfying mission deltaV)
%
=============================================================
============
else
Vmission = input('enter u r vmission value in m/s');
 end
mpl = input('enter payload mass in kg'); % [kg] payload mass
Isp =  input('enter specific impulse row matrix for each stage(sec)') ; % [sec] specific
impulses for each stage
epsilon =  input('enter structural ratio row matrix  for each stage') ; % [-] structural
ratios for
N = length (Isp);
g0 = 9.80665;
C = g0*Isp; % [m/s] exhaust velocities for each stage



p = Lagr_NR(Vmission, C, epsilon); % Lagrange multiplier
if isnan (p) == 0 % checking p is a defined number
L = (1 + p*C)./(p*C.*epsilon); % [-] mass ratios for each stage
lambda = (L.*epsilon - 1)./(1 - L); % [-] payload ratios for each stage
m = zeros(N,1);
m_pl = mpl;
for k = N:-1:1 % calculation of step masses of each
m(k) = m_pl/lambda(k); % stage beginning with stage N
m_pl = m_pl + m(k);
end
%optimum stage configuration
dV = C.*log(L); % [m/s] optimal deltaV split of stages
m_s = epsilon.*m; % [kg] structural masses for each stage
m_p = m - m_s; % [kg] propellant masses for each stage
m01 = sum(m) + mpl; % [kg] gross lift-off mass
lambda_t = mpl/m01; % [-] overall payload fraction
```

```
% print results to the command window
fprintf('optimal deltaV split of stages:\n')
for j = 1:N
fprintf('%d\t %g\n', j, dV(j))
end
fprintf('\noptimal stage masses:\n')
for j = 1:N
fprintf('%d\t %g\n', j, m(j))
end
fprintf('\noptimal structural masses:\n')
for j = 1:N
fprintf('%d\t %g\n', j, m_s(j))
end
fprintf('\noptimal propellant masses:\n')
for j = 1:N
fprintf('%d\t %g\n', j, m_p(j))
end
fprintf('\ngross lift-off mass:\t')
fprintf('%g', m01)
fprintf('\ntotal payload ratio:\t')
fprintf('%g', lambda_t)
else
fprintf('Failed to find a solution.')
fprintf('\nPlease increase Isp, or decrease epsilon or payload mass.\n')
end

function p = Lagr_NR(Vmission, C, epsilon)
% Solution of Lagrange Multiplier, p by Newton-Raphson method.
p_0 = -1/(min(C.*(1 - epsilon)));
p = p_0; % initial guess
tol = 10; % error tolerance
V = Vmission - sum(C.*log((1 + p*C)./(p*C.*epsilon)));
while abs(V)>tol
V = Vmission - sum(C.*log((1 + p*C)./(p*C.*epsilon)));
dV = sum(C./p./(1 + p*C));
d = -V./dV;
p = p + d;
end
```


MATLAB R2015b - academic use
Command Window
enter 1 if u have vmission value or /t else enter any number180152
enter injection altitude in km 200
enter apoge altitude in km 200
enter perigee altitude in km 200
enter orbit inclination in degrees 28.5
enter initial altitude inkm 9.1
enter initial latitude in deg 28.5
enter initial speed wrt to ground152.4
lift- off T/W Ratio 1.8
enter payload mass in kg150
enter specific impulse row matrix for each stage(sec)[305;325;350]
enter structural ratio row matrix for each stage[0.15;0.14;0.12]
ans =
1
optimal deltaV split of stages:
1 1747.65
2 2574.85
3 3799.64
optimal stage masses:
1 1914.62
2 1136.2
3 476.947
optimal structural masses:
15:22
13-04-2019


MATLAB R2015b - academic use
Command Window
enter structural ratio row matrix for each stage[0.15;0.14;0.12]
ans =
1
optimal deltaV split of stages:
1 1747.65
2 2574.85
3 3799.64
optimal stage masses:
1 1914.62
2 1136.2
3 476.947
optimal structural masses:
1 287.192
2 159.068
3 57.2337
optimal propellant masses:
1 1627.42
2 977.134
3 419.714
gross lift-off mass: 3677.77
total payload ratio: 0.0407856>>
15:23
13-04-2019