Optimal Design Of $\bar{X}$ Control chart Using Orthogonal Arrays

A project report submitted in partial fulfillment of the requirements forthe award of the degree of

BACHELOR OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

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2021-2022

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ACKNOWLEDGEMENT

It Is A Privilege For Us To Submit Project Report On Optimal Design Of $\overline{X}$ Control Chart By Using Orthogonal Arrays Submitted To Mechanical Engineering Section In Partial Fulfillment For The Award Of Bachelor Of Technology In Mechanical Engineering From Anil Neerukonda Institute Of Technology And Sciences - Sangivalasa.

We Would Like To Express Our Profound Gratitude And Sincere Indebtedness To Our EsteemedGuide **MS. K. V. RUKMINI** Asst.Professor Dept. Of Mechanical Engineering, AnilNeerukonda Institute Of Technology And Sciences, For His Inevitable Support, Timely GuidanceAnd Time He Has Devoted Towards Us In Doing This Project.

We Intend To Express Our Sincere Thanks With Our Obedience To Our Principal **Prof. T.V.Hanumantha Rao**, Head Of The Mechanical Engineering Department **Prof. Dr. B . Naga Raju.** We Also Convey Our Sincere Thanks To Our Department Faculty And Non-Teaching StaffFor Their Kind Co-Operation And Motivation.

Finally We Thank One And All Who Directly As Well As Indirectly Helped Us In The CompletionOf This Project.

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Abstract

- Control charts are widely used in industry for monitoring and controlling manufacturing processes. They should be designed economically in order to achieve minimum quality control costs. The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The $\bar{x}$ control chart dominates the use of any other control chart technique if quality is measured on a continuous scale. In the present project, the economic design of the $\bar{x}$ control chart using Orthogonal Arrays has been developed to determine the values of the sample size(n), sampling interval(h), width of control limits(k) such that the expected total cost per hour is minimized. Orthogonal arrays are used to determine the three parameters i.e., n, h & k. The result is compared with the literature and it is found to be superior to the cost in literature.

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CHAPTER - 1

Introduction

Statistical process control

Statistical process control (SPC) is a powerful collection of statistical methods to the monitor and control of a process to ensure that it operates as its full potential to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the least possible waste. A major objective of statistical process control is to quickly detect the occurrence of assignable causes of process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured.

Control chart

Control chart is one of the widely used statistical process control (SPC) tools. It is used to statistically monitor the process through sampling inspection. It indicates whether the process is in-control or out-of-control. If any point falls within the upper control limit and lower control limit, the process is referred to as "in-control" where if the point falls outsidethe control limits, the process is referred to as "out-of-control". The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action may be taken before a large number of nonconforming products are manufactured.

Control chart tells us when to allow the process to continue or avoid unnecessary adjustments with machine and when to take the corrective action. On to same problem either on the material side or from the operator side it is quite possible that either targeted value $\bar{X}$ has changed, or process dispersion has changed. These changes must be reflected on the control chart so that the corrective action can be taken.

The following are the factors that change the mean of the process:

- Tool and die wear,
- Machine vibrations,
- Wear or minor failure of machine parts change of machine or process, and
- Change of machine or process.

The following are the factors that change the process dispersion:

- Deviation of depression,
- Carelessness of operator,
- New or inexperienced work, and
- Frequent resetting of machine.

In any production process, regardless of how well designed or carefully maintained certain amount of inherent or natural variability will always exist. This natural variability or "background noises the cumulative effect of many small" essentially unavoidable causes. In the framework of statistical quality control, this natural variability is often called a "Stable system of change causes". A process that is operating with only change causes of variation present is said to be in statistical control .in other words, the chance causes are an inherent part of the process.

Other kinds of variability may occasionally be present in the output of a process. This variability in key quality characteristics usually arises from three sources: improperly adjusted or control machines, operator errors, or defective raw material. Such variability is generally large when compared to be background noise, and it usually represents an unacceptable level of process performance. Refer to these sources of variability that are not part of the chance cause pattern as assignable causes or variation. A process that is operating in the presence of assignable cause is said to be an out of control process.

As illustrated in Fig. 1.1 a control chart consists of the following lines:

i) a centre line ,

ii) a upper control limit, and

iii) a lower control limit.

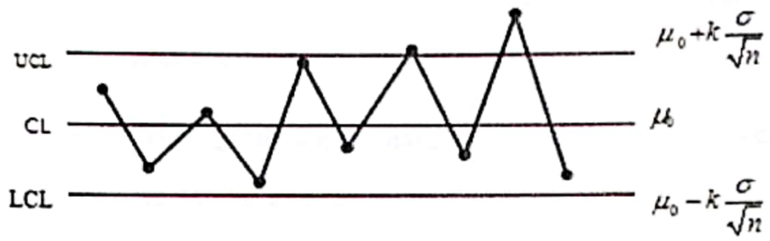

Fig. 1.1: $\bar{X}$ control chart

Types of control chart

The types of charts are often classified according to the type quality characteristics that they are supposed to monitor. Control charts can classify for variables chart and attributes charts.

Variable charts

The classical type of control chart is constructed by collecting data periodically and plotting it versus time. If more than one data value is collected at the same time, statistics such as the mean, range, median, or standard deviation are plotted. Control limits are addedto the plot to signal unusually large deviations from the centre line, and run rules are employed to detect other unusual patterns. Variable control charts are used for those quality characteristics which follow normal distribution, e.g. length, diameter etc.

- **$\bar{X}$ chart:** The $\bar{X}$ chart monitors the process location over time, based on the average of a series of observations, called a subgroup. This chart is used to plot the sample means in order to control the mean value of a variable.
- **R chart:** This chart is used to plot the sample ranges in order to control the variability of a variable. The range chart monitors the variation between observations in the subgroup over time.
- **S chart:** This chart is used to plot the sample standard deviations in order to control the variability of a variable.
- **S^2 chart:** This chart is used to plot the sample variances in order to control the variability of a variable.

Advantages of attribute control charts:

Attribute control charts have the advantage of allowing for quick summaries of various aspects of the quality of a product, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria. Thus, attribute charts sometimes the need for expensive, precise devices and time consuming measurement procedures. Also, this type of chart tends to be more easily understood by persons those are unfamiliar with quality control procedures: therefore, it may provide more persuasive (to management) evidence of quality problems.

Attribute charts

For attribute data, such as arise from pass or fail testing, the charts used most often plot either rates or proportions. When the sample sizes vary, the control limits depend on the size of the samples.

Attribute charts are a set of control charts specifically designed for attributes data. Attribute charts monitor the process location and variation over time in a single chart.

- **np chart**: For monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can either have this condition, or not have this condition.
- **p chart:** For monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can either have this condition, or not have this condition.
- **c chart:** For monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can have more than one instance of the condition.
- **u chart:** For monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can have more than one instance of the condition.

Advantages of variable control charts:

Variable control charts are more sensitive than attribute control charts. Therefore, variable control charts may alert us to quality problems before any actual "unacceptable" (as detected by the attribute chart) will occur. The variable control charts are leading indicators of trouble that will sound an alarm before the number of scraps increases inthe production process.

Other specialized control charts

Some of the specialized control charts are listed below:

1. Time–weighted charts,
2. Cumulative sum (CUSUM) control chart,
3. Moving average (MA) control chart, and
4. Exponential weighted moving average (EMWA) control.

1. Time-Weighted Charts

When data is collected one sample at a time and plotted on an individual's chart, the control limits are usually quite wide, causing the chart to have poor power in detecting out-of- control situations. This can be remedied by plotting a weighted average or cumulative sum of the data, not just the most recent observation. The average run length of such charts is usually much less than that of a simple $\bar{X}$ chart.

2. CUSUM Charts

A CUSUM Chart is a control chart for variables data which plots the cumulative sum ofthe deviations from a target. A V-mask is used as control limits. Because each plotted point on the CUSUM Chart uses information from all prior samples, it detects much smaller process shifts than a normal control chart would. CUSUM Charts are especially effective with a subgroup size of one. Run tests should not be used since each plotted point is dependent on prior points as they contain common data values.

3. Moving Average (MA) Chart

To return to the piston ring example, suppose we are mostly interested in detecting small trends across successive sample means. For example, we may be particularly concernedabout machine wear, leading to a slow but constant deterioration of quality (i.e., deviation from specification). The CUSUM chart described above is one way to monitor such trends, and to detect small permanent shifts in the process average. Another way is to use some weighting 8 schemes that summarize the means of several successive samples moving such a weighted mean across the samples will produce a moving average chart.

4. Exponentially-Weighted Moving Average (EWMA) Chart

The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, it could compute a geometric moving average.

Design of control chart

Design of a control chart involves the selection of three parameters, namely the sample size (n), the sampling interval (h) and the width of control limits (k). The selection of these three parameters is called the design of control chart.

Basically design of control chart is of three types explained below:

1. **Statistical design of control chart:** Since control chart is based on sampling inspection, it is always associated with two types of statistical errors namely Type-I error and Type-II error. These two errors cannot be completely eliminated since 100% inspection is not carried out. However, these two errors can be minimized which serves as the basic principle of statistical design of control chart.
2. **Economic design of control chart:** In economic design of control chart the objective is to reduce the total cost of maintaining the control chart as minimum as possible. It is used to determine the values of various design parameters i.e. sample size (n), sampling interval (h), and control limit coefficient (k) that minimizes total economic cost.

In all production processes, we need to monitor the extent to which our products meet specifications. In the most general terms, there are two "enemies" of product quality:

1. Deviations from target specifications, and
2. Excessive variability around target specifications.

The economic design of control charts is used to determine various design parameters that minimize total economic costs. The effect of production lot size on the quality of the product may also be significant. If the production process shifts to an out-of-control state at the beginning of the production run, the entire lot will contain more defective items. Hence, it is wiser to reduce the production cycle to decrease the fraction of defective items and, thus, improve output quality. On the other hand, reduction of the production cycle may result in an increase in costs due to frequent setups. A balance must be maintained so that the total cost is minimized. The production of quality goods depends upon the operating condition of the machine tools; however, the performance of machine tools depends upon the maintenance policy. It is assumed that the cost of maintaining the equipment increases with age; therefore, an age replacement strategy is needed to minimize the total cost of the system, which will simultaneously improve quality control and maintenance policy.

3. **Statistical economical design of control chart:** Statistical-economic design is basically a combination of statistical and economic design of control chart. In this type of design, the total cost of maintaining the control chart need to be minimized and at the same time Type-I and Type-II errors are not allowed to exceed their permissible level.

CHAPTER - 2

Literature Review

Design of control chart

Control charts are widely used in industry for monitoring and controlling manufacturing processes. Their design involve the selection of three parameters namely, the sample size (n), the sampling interval (h) and the width of control limits (k). **In case of economic design, these three control chart parameters are selected in such a manner that the total cost of controlling the process is the least.** The loss cost is expressed as a function of these three variables ***n*, *h*** and **k.**

Duncan (1956) proposed the first economic model for determining the three test parameters i.e. sample size (n), sample intervals (h) and width of control limit (k). For the $\overline{x}$ control chart that minimizes the average cost when a single out-of-control state (assignable cause) exists. Duncan's cost model includes the cost of sampling and inspection, the cost of defective products, the cost of a false alarm, the cost of searching for an assignable cause, and the cost of process correction.

TONG-YUAN KOOt and KENNETH E. CASE develops the economic design of an X-bar chart for monitoring continuous flow processes such as those encountered in refining, chemical processing and mining. The economic model permits one to optimally select the subgroup size, the sampling interval, and the control limit width based upon an economic criterion. This economic model is then applied to a realistic set of costs from the chemical industry. A sensitivity analysis is performed to illustrate the effects of incorrectly estimating the cost components of the model.

Cai et al. (2002) have proposed an economic model for the design of a control chart for a trended process. Traditional applications of the control charts are based on the assumption of process stability. But this is violated in many cases. The authors opine that the trended output resulting from a deteriorating factor like tool wear, material consumption, power consumption have to be interpreted differently. The researchers developed an economic model and tested the results.

Vijaya et al. (2007) has provided a simple approach to the robust economic design of control charts. Robust economic designs are capable of incorporating in them robustness corresponding to the ambiguity of cost in the cost and process parameters. Robust economic designs are of two types. One type considers the uncertainty in the estimation of cost and makes the design suitable for any scenario. The second type considers different discreet scenarios for a single process and makes the design robust for all possible scenarios. The researchers have introduced a simple statistic for the robust economic design process with many different scenarios. Simple genetic algorithm has been employed to optimize the test parameters.

Rashedi (2009) has proposed a new metaheuristic optimization algorithm based on law of gravity and mass interactions is introduced, namely gravitational search algorithm. To evaluate the GSA, they have examined it on a set of various standard benchmark functions.

Yu et al. (2010) have proposed an economic design of $\bar{X}$ control chart with multiple assignable causes. An economic design does not consider the statistical properties, for example, Type-I or Type-II error and average time to signal (ATC). To improve these issues, an economic statistical design of control charts has been developed under the consideration ofone assignable cause. However, there are multiple assignable causes in the real practice such as machine problem, material deviation, human errors; etc. In order to have a real application,this research will extend the original research from single to multiple assignable causes to establish an economic statistical model of $\bar{X}$ control chart.

Niaki et al (2011) presented an economic model using particle swarm optimization technique for the Economic - statistic model of MEWMA (multivariate exponentially weighted moving average) control charts. MEWMA control chart is used to monitor several correlated quality characteristics simultaneously and where we have to detect small deviations of the characteristics.

Gupta and Patel (2011) presented an economic design of $\bar{x}$ control chart using particle swarm optimization technique. In this work a computer program based on C language have been developed for economic design of the $\bar{X}$ control chart giving the optimum of n, h and k such that the expected total cost per hour is minimized.

Ganguly and Patel (2012) has proposed an economic design of $\bar{x}$ control chart using stimulated Annealing optimization technique. The mat lab codes are generated to minimize the loss cost by optimizing the design parameters like sample size n, sample frequency h, and control width k.

Taisir and Qasim (2013) implemented on The Performance of the gravitational search algorithm(GSA) are heuristic optimization algorithm based on Newton's law of gravity and mass interactions. In this work by fine tuning the algorithm parameters and transition functions towards better balance between exploration and exploitation. To assess its performance and robustness, compare it with that of genetic algorithms (G.A).

Dowlatshahi (2014) introduced GGSA i.e. A Grouping Gravitational Search Algorithm for data clustering. They adapt the structure of GSA for solving the data clustering problem, the problem of grouping data into clusters such that the data in each cluster share a high degree ofsimilarity while being very dissimilar to data from other clusters.

Abdul Sattar Safae (2015) presents a new framework to help the quality designer in order to react to uncertainty in the chart parameters. In this paper, economic statistical design (ESD) of the $\bar{x}$ control chart utilizing robust optimization approach that considers interval estimates of uncertain parameters is investigated. This approach gets popular in industrial practice because of its simplicity.

CHAPTER - 3

Chapter 3

Economic model for the design of control chart

The loss cost function has been formulated by Montgomery (1980) based on Duncan's (1956) economic model where the process is initially assumed to be in- control and normally distributed with mean μ_0 and variance σ^2. If sample size is n, for the $\bar{X}$ chart the centre line is at mean μ_0 and the two control limits are at

$$\mu_0 \pm k * \frac{\sigma}{\sqrt{n}} \tag{3.1}$$

Type-I error committed when control chart indicates that the process is out-of-control when it is actually in-control. It is also called as false alarm or producer's risk.

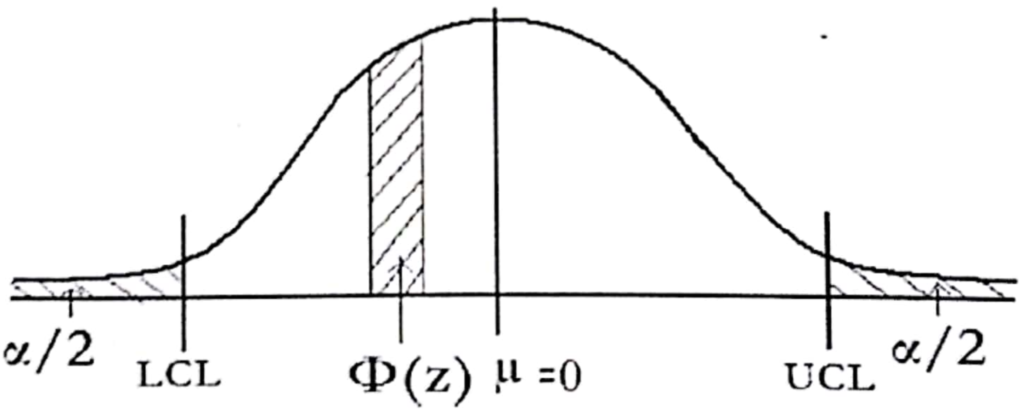

Fig. 3.1: Type-I Error

As illustrated in Fig. 3.1, the probability of committing Type-I error or the rate of falsealarm (α) is given by

$$\alpha = 2\int_{k}^{\infty} \psi(z)dZ \tag{3.2}$$

If the process mean has shifted by δ, but the area which is falling under the control limits is called as the probability of Type-II error or β error. It is also as called misdetection or consumer's risk.

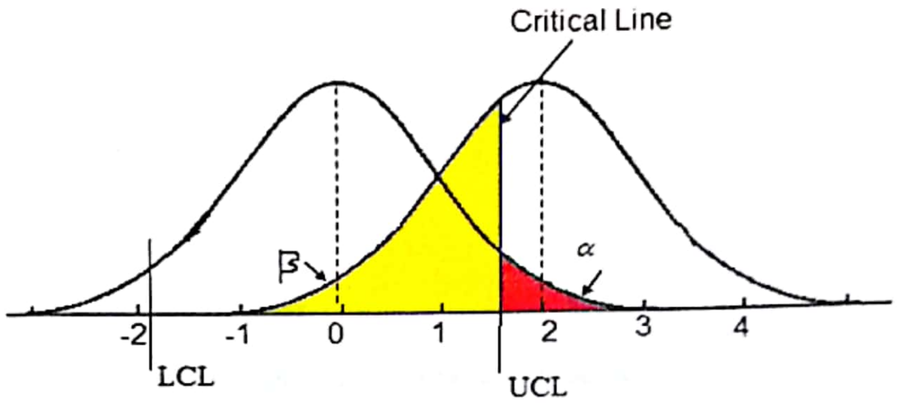

Fig. 3.2: Type II- Error

As illustrated in Fig. 3.2, when the assignable cause occurs, the probability that it willbe detected on any subsequent sample is power is calculated as follows

$$1\text{-}\beta = \int_{-\infty}^{(-k-\delta\sqrt{n}} \psi(z)dz + \int_{(k-\delta\sqrt{n}}^{\infty} \psi(z)dz \tag{3.3}$$

Notations:

a1 = Fixed component of sampling cost a2 = Variable component of sampling costa3 = Cost of finding an assignable cause

a_3^1 = Cost of investigating a false alarm

a_4 = Hourly penalty cost associated with production in out-of-control state

λ = Rate of occurrences of assignable cause per hour

g = Time to test and interpret the result per sample unit

τ = Expected time of occurrence of assignable cause since immediate past sampleD = Time

required to find an assignable cause and its elimination

n = Sample size

h = Sampling interval in hour

k = Width of control limitsα = Type – I error

β = Type – II error

μ_0 = Process mean for in-control processσ = Standard deviation

δ = Shift in process mean in multiple of σ

V_0 = the net income per hour of operation in the in-control state

V_1 = the net income per hour of operation in the out-of-control stateE (L) = Expected loss

per hour incurred by the process.

A production cycle consists of following four phases:

- The in-control phase,
- The out-of-control ,
- The time to take a sample and interpret the results i.e., gn, and
- The time to find the assignable cause i.e., D

In this work, the continuous model is assumed. The entire cycle isrepresented in Fig. 3.3.

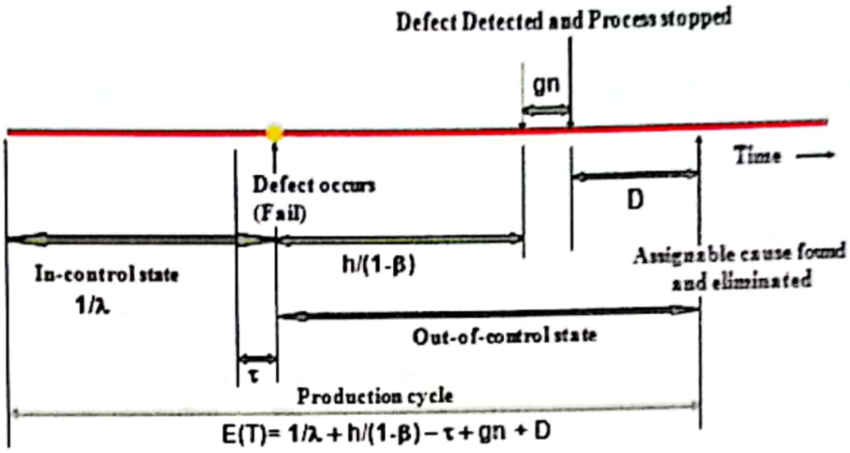

Fig. 3.3: Stages of a Production Cycle.

The four phases of a production cycle are as follows:

i. Assuming that the process begins in the in-control state and the assignable cause occurs at a rate of λ times per hour as per Poisson distribution, the time interval that the process remains in-control is an exponential random variable with a mean of $1/\lambda$. Thus, the expected in-control period is $1/\lambda$.

ii. If the number of samples required to produce an out-of-control signal when the process actually out-of-control is a geometric random variable with mean $1/(1-\beta)$, then the expected length of out-of-control period can be given by $[h/(1-\beta)] - \tau$.

iii. If g is the time required to take a sample of size 1 and interpret the results, then the total time for a sample size *n* will be gn.

iv. The time required to identify and remove the assignable cause following the signal is assumed to be a constant D.

Thus, the expected length of a cycle is

$$E(T) = \frac{1}{\lambda} + \left[\frac{h}{1-\beta} - \tau + gn + D\right] \quad (3.4)$$

When the process mean shifts, and the probability of its detection will be 1− β where β is Type-II error. Thus, the expected number of samples taken before the detection of shift will be 1/ (1− β). If the assignable cause occurs in a sample, then the expected initial time lag in that sample will be

$$\tau = \frac{\left(1-(1+\lambda h)e^{-\lambda h}\right)}{\lambda(1-e^{-\lambda h})} \quad (3.5)$$

The expected number of false alarms generated during a cycle is α times the expected numbers of samples taken before the shift, or

$$s = \frac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}} \quad (3.6)$$

If a' is cost of investigating a false alarm, total cost due to false alarm

$$\frac{a_3^1 \alpha e^{-\lambda h}}{1-e^{-\lambda h}} \quad (3.7)$$

Let a_1 and a_2 are fixed and variable components of sampling cost. Expected number of samples per cycle is E(T)/h. Thus, total cost of taking samples will be (a_1 + a_2 n) * E(T)/h .If V_0 and V_1 are the net income per hour of operation while the process is in-control and out-of-control respectively and a_3 is cost of finding an assignable cause, the expected total net income per cycle will be

$$E(C) = V_0(\frac{1}{\lambda}) + V_1(\frac{h}{1-\beta} - \tau + gn + D) - a_3 - \frac{a_3^1 \alpha e^{-\lambda h}}{1-e^{-\lambda h}} - \frac{(a_1 + a_2 n)}{h} \quad (3.8)$$

Expected net income per hour is E (A) = E(L)/E(T)

$$E(A)=V_0'-\frac{a_1+a_2n}{h}-\frac{\left(a_4\left(h\big/_{(1-\beta)}-\tau+gn+D\right)+a_3+a_3^1\alpha\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\right)}{\frac{1}{\lambda}+\left(h\big/_{(1-\beta)}-\tau+gn+D\right)}$$

(3.9)

Let the hourly penalty cost of producing with out-of-control process = V_0 □ V_1 □ a_4

$$E(A)=V_0'-\frac{a_1+a_2n}{h}-\frac{\left(a_4\left(h\big/_{(1-\beta)}-\tau+gn+D\right)+a_3+a_3^1\alpha\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\right)}{\frac{1}{\lambda}+\left(h\big/_{(1-\beta)}-\tau+gn+D\right)} \quad (3.10)$$

Or

$$E(A)=V_0-E(L) \quad (3.11)$$

Where

$$E(L)=\frac{a_1+a_2n}{h}-\frac{\left(a_4\left(h\big/_{(1-\beta)}-\tau+gn+D\right)+a_3+a_3^1\alpha\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\right)}{\frac{1}{\lambda}+\left(h\big/_{(1-\beta)}-\tau+gn+D\right)} \quad (3.12)$$

The expression E(L) represents the expected loss per hour incurred by the process and it is a function of three variables *n*, *h* and *k*. Maximizing the expected net income per hour E(A) is equivalent to minimizing E(L).

CHAPTER - 4

Chapter 4

Optimization Technique By Using Orthogonal Arrays

INTRODUCTION

The development of Taguchi's method is based on orthogonal arrays (OAs) that have a profound background in statistics. Orthogonal arrays were introduced in the 1940s and have been widely used in designing experiments. They provide an efficient and systematic way to determine control parameters so that the optimal result can be found with only a few experimental runs. This chapter briefly reviews the fundamental concepts of OAs, such as their definition, important properties, and constructions.

Definition of Orthogonal Array

Let S be a set of s symbols or levels. A matrix A of N rows and columns with entries from S is said to be an OA with s levels and strength t ($0 \leq l \leq k$) if in every $N \times t$ sub array of A, each t - tuple based on S appears exactly the same times as a row. The notation OA (N, k, s, t) is used to represent an OA.

To help readers understand the OA definition, orthogonal array OA(27, 10, 3, 2), which has 27 rows and 10 columns. Each entry of the array is selected from a set S={1,2, 3}. Thus, this is a three-level OA. Pick any arbitrary two columns (t=2) and one may see nine posible combinations as a row: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3). It can be simply verified that each combination has the same number of occurrences as a row, i.e., three times. This is the meaning of "orthogonal" in the definition, which ensures a balanced and fair selection of parameters in all possible combinations.

When this OA is used to design experiments, the 10 columns represent 10 parameters that need to be optimized. For each column, the entries 1, 2, and 3 denote three specific statuses or levels that an optimization parameter may select from. Note that for different optimization parameters, the levels 1, 2, and 3 may correspond to different numerical values. For example, if the optimization range for the parameter 1 is [0, 1], the corresponding values for levels (1,2, 3) could be (0.25, 0.5, 0.75).In contrast, if the optimization range for the parameter 2 is [-1, 0], the corresponding values for levels (1, 2, 3) change to (-0.75,-0.5,-0.25). Therefore, the corresponding values for the levels depend on the parameters and vary in different applications.

Each row of the OA describes a certain combination of the levels for these 10 parameters. For example, the first row means that all parameters take the level 1. The second row means that parameters 2 and 8 take level 1; parameters 1, 3, 4, 5, and 9 take level 2; and parameters 6, 7, and 10 take level 3. Once each parameter is assigned to a corresponding level value, one can conduct the experiment and find the output result. It is important to point out that the 27 rows of the OA indicate that 27 experiments need to be carried out per design iteration.

IMPORTANT PROPERTIES

The OA's have several important properties. For the brevity of the book, three fundamental characteristics are highlighted here, which are useful for Taguchi's method discussed in later chapters. The following are the three fundamental characteristics:

- The first one is the fractional factorial characteristic.

Using the above example that includes 10 parameters where each has three levels, one noticesthat a full factorial strategy needs to conduct 59049 experiments. In contrast, if one uses the OA to design such experiments, only 27 experiments are needed. After a simple analysis and processing of the output results from the 27 experiments, an optimum combination of the parameter values can be obtained. Although the number of experiments is dramatically reduced from 59 049 to 27, statistical results demonstrate that the optimum outcome obtained from the OA usage is close to that obtained from the full-factorial approach.

- The second fundamental property of the OA is that all possible combinations of up to t parameters occur equally, which ensures a balanced and fair comparison of levels for any parameter and any interactions of parameters.

A quick examination reveals that for each parameter (column), levels 1, 2, and 3 have nine times of occurrences. Thus, all possible levels of a parameter are tested equally. A similar property applies for the combination of any two parameters. Therefore, the OA approach investigates not only the effects of the individual parameters on the experiment outcome, but also the interactions of any two parameters.

In general, one could increase the strength of the OA to consider interactions between more parameters. However, the larger the strength is, the more rows (experiments) the OA has. The OAs used in this book have a strength of 2, which is found to be efficient for the problems considered.

- The third useful property of OAs is that any $N \times k'$ sub array of an existing OA (N, k, s, t) is still an OA with a notation $OA(N, k', s, t')$, where $t' = \min\{k', t\}$. In other words, if one or more columns are deleted from an OA, the resulting array is still an OA but with a smaller number of parameters.

For example, if we delete the last two columns we can obtain an OA(27, 8, 3, 2). This property is especially useful when selecting an OA from an existing OA database. If an OA with a certain number of columns (k') cannot be found in the database, one can choose an OA with a larger number of columns ($k > k'$) and manually delete the redundant ($k-k'$) columns to obtain the desired OA.

This chapter presents the optimization technique using Taguchi's method. A fundamental implementation procedure is introduced first and several optimization problems are used as examples to demonstrate its validity. Based on this development, several improvement techniques are proposed to enhance the optimization performance.

IMPLEMENTATION PROCEDURE

To illustrate the implementation procedure, a 10-dimensional Rastigrin function is used as

$$f(x) = \sum_{i=1}^{10}[x_i^2 - 10\cos(2\pi x_i) + 10], \quad -9 < x_i < 8$$

The function has a global minimum 0 when all xi = 0.The 10 input parameters (xi, *i* =1,….,10) are optimized to find the global minimum. The search range is set to [-9, 8] to obtain an asymmetrical optimization space. The optimization procedure of the first iteration is explained in detail, and the procedure of the remaining iterations is similar to that of the first iteration.

Problem Initialization

The optimization procedure starts with the problem initialization, which includes the selection of a proper OA and the design of a suitable fitness function. The selection of an OA(N, *k*, s, *t*) mainly depends on the number of optimization parameters. There are 10 parameters that should be optimized. Thus, the OA to be selected must have 10 columns ($k = 10$) to represent these parameters. To characterize the nonlinear effect, three levels ($s = 3$) are found sufficient for each input parameter. Usually, an OA with a strength of 2 ($t = 2$) is efficient for most problems because it results in a small number of rows in the array. In summary, an OA with 10 columns, 3 levels, and 2 strengths is needed.

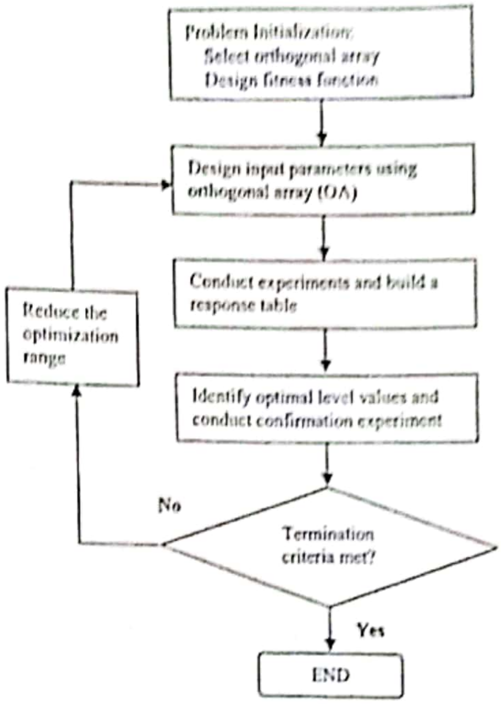

Fig. 4.1: Flow chart of Orthogonal Array

After searching the online OA database an OA(27, 13, 3, 2) is found to be available. To be consistent with this problem, the first 10 columns of the OA are retained whereas the rest (3 columns) are deleted. Hence, an OA(27, 10, 3, 2) is obtained for the optimization process.

The fitness function is chosen according to the optimization goal. In this optimization example the fitness function is selected to be the same as the Rastigrin function:

$$E(L) = \frac{a_1 + a_2 n}{h} - \frac{\left(a_4 \left(\frac{h}{(1-\beta)} \right)^{-\tau + gn + D} + a_3 + a_3^1 \alpha \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \right)}{\frac{1}{\lambda} + \left(\frac{h}{(1-\beta)} \right)^{-\tau + gn + D}}$$

where the fitness can be considered as the difference between the optimization goal (0 value) and the obtained value from the current inputs x. The smaller the fitness value, the better the match between the obtained value and the desired one.

Design Input Parameters Using OA

Next, the input parameters need to be selected to conduct the experiments. When the OA is used, the corresponding numerical values for the three levels of each input parameter should be determined.

In the first iteration, the value for level 2 is selected at the center of the optimization range. Values of levels 1 and 3 are calculated by subtracting/adding the value of level 2 with a variable called level difference (LD). The level difference in the first iteration (LD,) is determined by the following equation:

$$LD_1 = \frac{\max - \min}{\text{number of levels} + 1} = \frac{8-(-9)}{3+1} = 4.25,$$

where max is the upper bound of the optimization range and min is the lower bound of the optimization range. Thus, the three levels are uniformly distributed in the optimization region. With the use of each entry of the OA can be converted into a corresponding level value of the input parameter where n indicates the nth element, the subscript 1 indicates the first iteration, and the superscript m indicates the level 1, 2, or 3.

Identify Optimal Level Values and Conduct Confirmation Experiment

When the optimal levels are identified, a confirmation experiment is performed using the combination of the optimal levels identified in the response table. This confirmation test is not repetitious because the OA-based experiment is a fractional factorial experiment, and the optimal combination may not be included. The fitness value obtained from the optimal combination is regarded as the fitness value of the current iteration.

Reduce the Optimization Range

To reduce the optimization range for a converged result, the LD, is multiplied with a reduce rate (rr) to obtain LD i+1 for the (*i*+1)th iteration:

$$LD_{i+1} = rr \cdot LD_i = rr^i \cdot LD_1 = RR(i) \cdot LD_1,$$

where RR(i) is called reduced function. When a constant rr is used, RR(i) = r. The value of rr can be set between 0.5 and 1 depending on the problem. The larger rr is, the slower the convergence rate. In this Rastigrin function optimization, rr is set to 0.6.

Check the Termination Criteria

When the number of iterations is large, the level difference of each element becomes small. Hence, the level values are close to each other and the fitness value of the next iteration is close to the fitness value of the current iteration. The following equation may be used as a termination criterion for the optimization procedure:

$$\frac{LD_i}{LD_1} < \text{converged value}.$$

Usually, the converged value can be set between 0.001 and 0.01 depending on the problem. The iterative optimization process will be terminated if the design goal is achieved or if it is satisfied.

CHAPTER - 5

Results and Comparison

Numerical Examples

1. For testing the effectiveness of orthogonal array in economic design of control chart, a numerical example earlier solved by simulated annealing (2012) was considered.

A manufacturer produces a nonreturnable glass bottles for packaging a carbonated soft drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. The manufacturer has used $\bar{X}$ control charts for process surveillance for some time. These control charts have been designed with respect to statistical criteria. However, in an effort to reduce costs, the manufacturer wishes to design an economically optimum $\bar{X}$ control chart for the process.

Based on an analysis of quality control technicians' salaries and the costs of the test equipment, it is estimated that the fixed cost of taking a sample is \$1. The variable cost of taking a sample is estimated to be \$0.01 per bottle. It takes approximately 1 minute (0.0167 h) to measure and record the wall thickness of the bottle. The process is subject to several different types of assignable causes. However when the process goes out-of-control, the magnitude of shift is approximately two standard divisions. Process shift occurs at random with a frequency of about one every 20h operation. Thus the exponential distribution with parameter $\lambda = 0.05$ is a reasonable model of the run length in control. The average time required to investigate an out-of-control signal is 1h. The cost of investigating an action signal that results in the elimination of the assignable cause is \$25, while the cost of investigating a false alarm is \$50. The manufacturer estimates that the penalty cost of operating in the out-of-control state for one hour is \$100. An economic model for the X control chart had to be designed.

Data given: a1 =\$1, a2 =\$0.10, a3 =\$25, a 1 = \$50, a4 =\$100, λ =0.05, d =2.0, g=0.0167, D=1.0 and s =2.

In the present work, the loss cost function has been minimized by using Orthogonal Arrays. Out of three variables, sample size (*n*) is interfere where as other i.e. sampling interval (*h*) and width of control limits (*k*) take real values on continuous scale. The search domain selected for searching the optimum solution are 1 to 15 for *n*, 0.1h to 1.0 *h* for *h* and 0.1 to 5 for *k*. The optimum values of *h* and *k* along with corresponding minimum values of expected loss cost E(L) obtained for various integer values of *n* varying from 1 to 15 have been listed.

2. A plant, located in central Taiwan, produces grape juice, which is contained in glass bottles. The target quantity of grape juice is 200 cm_3 for each bottle. In the production process, the grape juice is inserted into twelve bottles at a time, and the twelve bottles of juice will be packed in a box later. Before the twelve bottles of grape juice are packed, the inspector samples the first four bottles to check whether the quantity of grape juice for each bottle is 200 cm3. An $\bar{X}$ chart is applied to monitor the process of insertion. The inspector complains that it is so frequent to sample the bottles. Therefore, the plant will conduct the economic design of the $\bar{X}$ chart such that the sampling related costs may be minimized. Based on an analysis of inspector's salaries and the costs of test equipment, it is estimated that the fixed cost of taking a sample is \$1 (i.e. a_1=1). The estimated variable cost of sampling is about \$0.10 per bottle (i.e. a_2 = 0.10) and it takes ~ 1 minute (g = 0.0167 hour) to measure and record the quantity of a bottle. Process shifts occur at random with a frequency of about one every 20 h of operation (λ = 0.05). On the average, when the process goes out of control, the magnitude of the shift is SDs (δ = 2.0). The average time required to investigate an out-of - control signal is 1 h (D = 1). The cost of investigating an action signal that results in the elimination of an assignable cause is $25, while the cost of investigating a false alarm is $50 (i.e. a3 = 25 and a5 = 50). The manufacturer estimates that the penalty cost of operating in the out-of-control state for 1 h is $100 (i.e. a4 = 100). Previous data indicate that the skewness and kurtosis coefficients of the quantity of content of a bottle of packed grape juice are 0.4836 and 3.3801, respectively, which may be described by the Burr distribution with c = 3 and q = 6. The recent 60 successive boxes are viewed as a random sample from a multivariate distribution. The first four bottle quantity averages are μ = [190, 202, 214, 208], and the covariance matrix of these dimension is

$$\mathbf{V} = \begin{bmatrix} 5.92 & 5.69 & 4.57 & 4.39 \\ 5.69 & 6.69 & 5.01 & 5.77 \\ 4.57 & 5.01 & 7.48 & 5.23 \\ 4.39 & 5.77 & 5.23 & 6.67 \end{bmatrix}.$$

3. To illustrate this procedure, let $\delta = 2$, $\lambda = 0.01$, $M = \$100$, $e = 0.05$, $D = 2$, $T = \$50$, W = $25, b = $ 0.50, and c = $0.10

4. An example will be used to illustrate the use of the model and to learn about its robustness to inaccuracies in input parameters and costs. The example comes from the chemical industry where a polymer commodity is being produced. The most important measurable characteristic of the polymer is its viscosity which is typically targeted at a value of 70.0, but sometimes takes swings which it is desirable to detect using a control chart. The standard deviation is approximately 0·8 and remains relatively constant.

Experience shows that the process, once it is stabilized after a grade change, averages 50 h of continuous operation before the mean shifts, causing an out - of - control condition. The distribution of time to process shift follows reasonably well the exponential distribution, and has some very short runs in-control, as well as a few very long runs. The exponential parameter is therefore estimated to be λ = 1/50= 0·02. A shift of ± 1·6 viscosity units is considered a typical magnitude shift, so " is estimated to be 2·0. Off-target polymer must be monitored closely, segregated, stored, and reprocessed or blended (seldom is it scrapped), causing significant additional work and encumbrance of plant and equipment. The reduction in hourly income attributed to an assignable cause is M = us $4000 per hour.

The time for a sample to be taken, transmitted to the laboratory, and the results phoned back to the process control room is e = 1·25h. Each time an assignable cause indication occurs, steps taken to find the cause(s) are quite costly. Substantial manpower may be required, and the production rate is often decreased until the cause is found. The time necessary to find an assignable cause after a point has fallen outside control limits averages D = 2 h. The estimated cost of finding an existing assignable cause is an average of W=us$1000; since the search continues extensively when the OOC indication is really a false alarm, its cost is estimated to be double, or T = us$2000. The cost of maintaining the control chart and plotting an average is estimated to be b = us $20 per subgroup, and the cost per unit of sampling and analytical analysis is c = us $20 per sample.

Table 5.1. Given data of solved examples

Q.no	Journal & Author	a1	a2	a3	A3	a4	g	δ	D	λ
1.	A. Ganguly and S. K. Patel	1	0.1	25	50	100	0.0167	2	1	0.05
2.	Chao - yu chou, Chury - Ho - Chun	1	0.1	25	50	100	0.0167	2	1	0.05
3.	Acheson J. Duncan	0.5	0.1	25	50	100	0.05	2	2	0.01
4.	Tong – yuan – kao KENNETHE. CASE	20	20	1000	2000	4000	1.25	2	2	0.02

Calculations of Solved Example

Given data : $a_1 = 1$, $a_2 = 0.1$, $a_3 = 25$, $\delta = 2.0$, D=1.0, $a_4 = 100, a_{3'} = 50, \lambda = 0.05, g = 0.0167, \sigma = 2$

Limits :

n : 1 to 15

h: 0.1h to 1h

k:0.1 to 5

Orthogonal array:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 2 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$L_D = \max - \min / (Number$ of levels +1)

1*st* Iteration:

for n:

n_0 = midvalue - L_D

n_1 = midvalue

n_2 = midvalue + L_D

L_D = (15-1)/(3+1) = 3.5

$n_0 = 8 - 3.5 = 4.5$ □ 5

n_1 = (1+15)/2 =8

n_2 = 8+3.5 =11.5

for h:

h_1=(0.1 +1)/2 =0.55

$h_0 = h_1$-LD

$LD = (1 - 0.1) / 4 = 0.225$

h_0 = 0.55 -0.225=0.325

h_2 =0.55+0.225=0.775

for k:

k_1=(0.1+5)/2=2.55

LD =(5-0.1)/4=1.225

k_0=2.55-1.225=1.325

k_2 =2.55+1.225=3.775

$$E(L) = (a_1 + a_2 n) / h + [a_4(h/(1-\beta) - \tau + gn + D) + a_3 + a_3^1 \alpha e^{-\lambda h} / 1 - e^{-\lambda h}] / (1/\lambda + (h/(1-\beta) - \tau + gn + D))$$

α=2(1-ϕ(k))

$\beta = \phi(k - \delta\sqrt{n}) - \phi(-k - \delta\sqrt{n})$

For 1st row:

α=2(1-ϕ(k))

$n = 5, h = 0.325, k = 1.325$

α=2(1-ϕ(0.32))

α=ϕ(1.325)=0.90658 $\Rightarrow \alpha = 0.18684$

$\beta = \phi(1.325 - 2\sqrt{5}) - \phi(-1.325 - 2\sqrt{5})$

β=1 − ϕ(3.167)-(1-ϕ(5.79))

β=1 − 0.99921 − 1 + 1

1 − β=0.99921

$\tau = (1-(1+\lambda h)e^{-\lambda h}) / \lambda(1-e^{-\lambda h}); \lambda h = 0.01625$

$\tau = (1-(1+0.01625)e^{-0.01625}) / 0.05(1-e^{-0.01625}) \Rightarrow \tau = 0.17$

$E(L) = (1+(0.1)5)/0.325$ + (100(0.325/0.99921) -0.17+0.0835+1)+25+570.2)/

(20+(0.325/0.99921)-0.17+0.0835)

E(L)=4.61 + 32.79=37.4

for 2nd row:

n=5 h=0.55 k=2.55

α=2(1-ϕ(k)) = 2(1-ϕ(2.55))

α=2(1 − 0.99461) ⇒ $\alpha = 0.01078$

β= $\phi(2.55 - 2\sqrt{5}) - \phi(-2.55 - 2\sqrt{5})$

β=$\phi(-1.92) - \phi(-7.02)$

= 1-$\phi(1.92) - (1 - \phi(7.02))$

=1-0.97257-1+1

1-β=0.97257; $\lambda h = 0.0275$

$\tau = (1-(1+0.0275)e^{-0.0275})/0.05(1-e^{-0.0275}) \Rightarrow \tau = 0.33038$

$E(L) = (1+0.1(5))/0.55) + ([100((0.55/0.97257) - 0.33038 + 0.0835 + 1] + 25 + 0.539(35.764))/$
$(20 + (0.55)/0.97257) - 0.33038 + 0.0835 + 1))$

$E(L) = 2.727 + (131.8649 + 25 + 19.276)/21.318$

$E(L) = 2.727 + 8.2625 = 10.9$

for 3rd row:

n=5,h=0.775,k=3.775

α=2(1-ϕ(k)) = 2(1-ϕ(3.775))

α=2(1 − 0.99992) ⇒ $\alpha = 0.0016$

β= $\phi(3.775 - 2\sqrt{5}) - \phi(-3.775 - 2\sqrt{5})$

β=$\phi(-0.69713) - \phi(-8.2471)$

β=1 − $\phi(0.69713) - (1-1)$

β=1 − 0.75803 ⇒ $1 - \beta = 0.75803$

$\lambda h = 0.03875$

$\tau = (1-(1+0.03875)e^{-0.03875})/0.05(1-e^{-0.03875})$

$\tau = (1-(1.03875)(0.96199))/0.05(1-0.96199)$

$\tau = 0.385628$

$E(L) = (1+0.1(5))/0.775) + [100(1.0223 - 0.385628 + 1.0835) + 25 + 0.2024]/$
$[20 + (1.0223 - 0.385628 + 1.0835)]$

$E(L) = (1+0.5/0.775) + [100(1.0223 - 0.385628 + 1.0835) + 25.2024]/$
$[20 + (1.0223 - 0.385628 + 1.0835)]$

$E(L) = 2.0134 + 197.2412/21.720$

=2.0134+9.0639=11.0773

for 4th row:

n=8,h=0.325,k=2.55

α=2(1-ϕ(k)) = 2(1-ϕ(2.55))

α=2(1 − 0.99461) $\Rightarrow \alpha = 0.01078$

β=$\phi(k-\delta\sqrt{n})-\phi(-k-\delta\sqrt{n})$

β= $\phi(2.55-2\sqrt{8})-\phi(-2.55-2\sqrt{8})$

β=$\phi(-3.106)-\phi(-8.206)$

β=$1-\phi(3.106)$

β=$1-0.99906 \Rightarrow 1-\beta = 0.99906$

$\lambda h = 0.01625$

$\tau = 0.17$

$E(L) = (1+0.1(8))/0.325) + [100(0.3253-0.177+0.1336+1)+25+0.539(61)]/$

$[20+(0.3253-0.17+1.1336)]$

$E(L) = 5.538 + 186.769/21.2889$

=5.538+8.7=14.238

for 5th row:

n=8,h=0.55,k=3.775

α=2(1-ϕ(k)) = 2(1-ϕ(3.775))

α=2(1 − 0.99992) $\Rightarrow \alpha = 0.0016$

β= $\phi(3.775-2\sqrt{8})-\phi(-3.775-2\sqrt{8})$

$1-\beta = 0.97062$

$\tau = 0.33038$

$E(L) = (1+0.1(8))/0.55) + [100(0.566648-0.2737+0.1336+1)+25+0.28692]/$

$[20+(0.566648-0.2737+0.1336)]$

$E(L) = 3.272 + 167.94172/21.426548$

=3272+7.522=10.794

for 6th row:

n=8,h=0.775,k=1.325

α=2(1-ϕ(1.325))

$\alpha=2(1-0.90824) \Rightarrow \alpha = 0.18352$

$\beta= \phi(1.325-2\sqrt{8})-\phi(-1.325-2\sqrt{8})$

$\beta=\phi(-4.33)-\phi(0.981)$

$\beta=0 \Rightarrow 1-\beta=1$

$\tau = 0.3849$

$E(L)=(1+0.1(8))/0.775)+[100(0.775-0.3849+0.1336+1)+25+9.176(25.309)]/$

$[20+(0.775-0.3849+0.1336+1)]$

$E(L)=2.3225+(177.3602+232.235)/21.52$

=21.3557

for 7th row:

n=12,h=0.325,k=3.775

α=2(1-ϕ(3.775))

$\alpha=2(1-0.99992) \Rightarrow \alpha = 0.00016$

$\beta= \phi(3.775-2\sqrt{12})-\phi(-3.775-2\sqrt{12})$

$\beta=\phi(-3.15)-\phi(-10.7)$

$1-\beta=0.99918$

$\tau = 0.17$

$E(L)=(1+0.1(12))/0.325)+[100(0.3252-0.17+0.2004+1)+25+0.008(61.039)]/$

$[20+(0.3252-0.17+0.2004+1)]$

$E(L)=6.7693+161.0148/21.3556$

=14.31

for 8th row:

n=12,h=0.55,k=1.325

α=2(1-ϕ(k)) = 2(1-ϕ(1.325))

$\alpha=2(1-0.90824) \Rightarrow \alpha = 0.18352$

$\beta = \phi(1.325 - 2\sqrt{12}) - \phi(-1.325 - 2\sqrt{12})$

$\beta = 0$

$1 - \beta = 1$

$\lambda h = 0.55 * 0.05 = 0.275$

$\tau = 0.33038$

$E(L) = (1 + 0.1(2)) / 0.55) + [100(0.55 - 0.33038 + 0.20041 + 1) + 25 + 329.1450] /$

$[20 + (0.55 - 0.33038 + 0.20041 + 1)]$

$E(L) = 4 + 496.142 / 21.42002$

$= 27.16$

for 9th row:

n=12,h=0.775,k=2.55

$\alpha = 2(1 - \phi(k)) = 2(1 - \phi(2.55))$

$\alpha = 0.01078$

$\beta = \phi(2.55 - 2\sqrt{12}) - \phi(-2.55 - 2\sqrt{12})$

$1 - \beta = 1$

$\lambda h = 0.0385628$

$\tau = 0.385628$

$E(L) = (1 + 1.2) / 0.775) + [100(0.775 - 0.385628 + 1.20041) + 25 + 13.64] /$

$[20 + (0.775 - 0.385628 + 1.20041 - 0.3385628)]$

=2.8387+197.612/21.589772

=2.8387+9.15=11.9857

on sorting in ascending order

$n_1=8$	$h_1=0.55$	$k_1=3.775$
$L_D=0.6*L_{D1}$	$L_D=0.6*L_{D1}$	$L_D=0.6*L_{D1}$
$L_D=0.6*3.5=2.1$	$L_D=0.6*0.225=0.135$	$L_D=0.6*1.225=0.735$
$n_0=8-2.1=5.9$	$h_0=0.415$	$k_0=3.04$
$n_2=8+2.1=10.1$	$h_2=0.685$	$k_2 = 4.51$

for 1st row:

n=5,h=0.415,k=3.04

α=2(1-ϕ(k)) = 2(1-ϕ(3.04))

$\alpha = 2(1-99882) = 0.00236$

β= $\phi(3.04\text{-}2\sqrt{5}) - \phi(\text{-}3.04 - 2\sqrt{5})$

β=1-$\phi(1.43) - \phi(7.512)$

$1-\beta = \phi(1.43)$=0.92364

$\lambda h = 0.02075$

$\tau = (1-(1+\lambda h)e^{-\lambda h}) / \lambda(1-e^{-\lambda h})$

$\tau = (1-(1.02075)(0.9794637) / 0.05(1-0.9794637)$

$\tau = 0.2067$

$E(L) = (1+0.1(5)/0.415) + [100(0.415/0.92364 - 0.2067 + 1.10835) + 25 + 5.627] /$

$[20 + (0.4493 + 1.05 - 0.2067)]$

=3.8544+7.75432=11.6084

for 2nd row:

n=6 h=0.55 k=3.775

α=2(1-ϕ(k)) = 2(1-ϕ(3.775))

α=2(1−0.99992) $\Rightarrow \alpha = 0.00016$

β= $\phi(3.775 - 2\sqrt{6}) - \phi(\text{-}3.775 - 2\sqrt{6})$

β = 1-ϕ(1.12)

1-β=$\phi(1.12) = 0.86864$

$\lambda h = 0.55 * 0.05 = 0.0275$

$\tau = (1-(1+\lambda h)e^{-\lambda h}) / \lambda(1-e^{-\lambda h}) = 0.2737$

$E(L) = (1+0.6)/0.55) + ([100(0.63317 - 0.273710 + 0.1002 + 1] + 25 + 0.2869 / (20 + 1.45967)$

$E(L) = 2.909 + 171.2539 / 21.45967$

$E(L) = 2.909 + 7.980 = 10.889$

for 3rd row:

n=6,h=0.685,k=4.51

α=2(1-ϕ(4.51)) = 2(1-ϕ(3.775))

α=2(1 − 0.99992) $\Rightarrow \alpha = 0$

β= $\phi(4.51 - 2\sqrt{6}) - \phi(-4.51 - 2\sqrt{6})$

β=ϕ(0.388979)

β=0.65173 $\Rightarrow 1 - \beta = 0.34827$

$\lambda h = 0.05 * 0.685 = 0.03425$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = (1 - (1.03425)(0.966329) / 0.05(1 - 0.966329)$

$\tau = 0.3405$

$E(L) = (1 + 0.6) / 0.685) + [100(1.966 - 0.3405 + 1.1002) + 25] /$

$[20 + (1.966 - 0.3405 + 1.1002)]$

$E(L) = 2.3357 + 297.57 / 22.7257$

=2.3357+13.093=15.42

for 4th row:

n=8,h=0.415,k=3.775

α=2(1-ϕ(k))

$\alpha = 0.00016$

β=$\phi(k - \delta\sqrt{n}) - \phi(-k - \delta\sqrt{n})$

β= $\phi(3.775 - 2\sqrt{5}) - \phi(-3.775 - 2\sqrt{5})$

β=1 − ϕ(1.89)

$1 - \beta = 0.97062$

$\lambda h = 0.05 * 0.415 = 0.02075$

$\tau = 0.2067$

$E(L) = (1 + 0.1(8)) / 0.415) + [100(0.14275 - 0.2067 + 0.1336 + 1) + 25 + 0.381] / [20 + 1.35437]$

$E(L) = 4.337 + 160.827 / 21.3543$

=4.337+7.531=11.86

for 5th row:

n=8,h=0.55,k=4.51

α=2(1-ϕ(k))

$\alpha = 0$

β=$\phi(k-\delta\sqrt{n})-\phi(-k-\delta\sqrt{n})$

β= $\phi(4.51-2\sqrt{8})-\phi(-4.51-2\sqrt{8})$

β=$1-\phi(1.15)$

$1-\beta = 0.87493$

$\lambda h = 0.55 * 0.05 = 0.0275$

$\tau = 0.2737$

$E(L) = (1+0.1(8))/0.55) + [100(0.62862-0.2737+1.1336)+25+0]/[20+1.48852]$

$E(L) = 3.272 + 173.852/21.48852$

=11.362

for 6th row:

n=8,h=0.685,k=3.04

α=2(1-ϕ(k))

α=2(1-ϕ(3.04))

$\alpha = 0.00236$

$\beta = \phi(k - \delta\sqrt{n}) - \phi(-k - \delta\sqrt{n})$

$\beta = \phi(3.04 - 2\sqrt{8}) - \phi(-3.04 - 2\sqrt{8})$

$\beta = 1 - \phi(2.62)$

$1 - \beta = 0.99560$

$\lambda h = 0.03425$

$\tau = 0.3405$

$E(L) = (1 + 0.1(8)) / 0.685) + [100(0.6880 - 0.3405 + 1.1336) + 25 + 3.386] / [20 + 1.4811]$

$E(L) = 2.627 + 176.496 / 21.4811$

=10.8433

for 7th row:

n=10,h=0.415,k=4.51

α=2(1-ϕ(k))

α=2(1-ϕ(4.50))

$\alpha = 0$

$\beta = \phi(k - \delta\sqrt{n}) - \phi(-k - \delta\sqrt{n})$

$\beta = \phi(4.51 - 2\sqrt{10}) - \phi(-4.51 - 2\sqrt{10})$

$\beta = 1 - \phi(1.81)$

$1 - \beta = 0.96485$

$\lambda h = 0.02075$

$\tau = 0.2067$

$E(L) = (1 + 0.1(10)) / 0.415) + [100(0.4301 - 0.2067 + 1.167) + 25] / [20 + 1.3904]$

$E(L) = 4.819 + 7.668$

=12.487

for 8th row:

n=10,h=0.55,k=3.04

α=2(1-ϕ(k))

α=2(1-ϕ(3.04))

$\alpha = 0.00236$

β=$\phi(k-\delta\sqrt{n})-\phi(-k-\delta\sqrt{n})$

β= $\phi(3.04-2\sqrt{10})-\phi(-3.04-2\sqrt{10})$

β=1$-\phi$(3.28)

$1-\beta = 0.999948$

$\lambda h = 0.0275$

$\tau = 0.2737$

$E(L) = (1+0.1(10))/0.55)+[100(0.55026-0.2737+1.167)+25+4.232]/[20+1.44356]$

$E(L) = 3.636+173.588/21.44350$

E(L) =11.73

for 9th row:

n=10,h=0.685,k=3.775

α=2(1-ϕ(k))

α=2(1-ϕ(3.775))

$\alpha = 0.00016$

β=$\phi(k-\delta\sqrt{n})-\phi(-k-\delta\sqrt{n})$

β= $\phi(3.775-2\sqrt{10})-\phi(-3.775-2\sqrt{10})$

β=1$-\phi$(2.55)

$1-\beta = 0.99461$

$\lambda h = 0.03425$

$\tau = 0.3405$

$E(L) = (1+0.1(10))/0.685)+[100(0.68887-0.3405+1.167)+25+0.2295]/$

$[20+1.15152]$

$E(L) = 2.919+8.215$

=11.134

$3rd$ iteration

on sorting in ascending order

n_1=8	h_1=0.685	k_1=3.04
L_D=0.6*L_{D2}	L_D=0.6*L_{D2}	L_D=0.6*L_{D2}
L_D=0.6*2.1=1.26	L_D=0.6*0.135=0.081	L_D=0.6*0.735=0.441
n_0=8-1.26=6.74 □ 7	h_0=0.604	k_0=0.415
n_2=9.26 □ 7	h_2=0.766	$k_2 = 3.481$

For 1st row:

$n = 7, h = 0.604, k = 2.599$

α=2(1-ϕ(k))

α=2(1-ϕ(2.56))=2(1-0.99477)=0.01046

β= $\phi(2.599 - 2\sqrt{7}) - \phi(-2.599 - 2\sqrt{7})$

β=$1 - \phi(2.60)$

$1 - \beta$=0.99643

$\lambda h = 0.05 * 0.604 = 0.0302$

$\tau = (1-(1+\lambda h)e^{-\lambda h}) / \lambda(1-e^{-\lambda h}); \lambda h = 0.01625$

$\tau = (1-(1.0302)(0.9702514) / 0.05(0.029748) \Rightarrow \tau = 0.3004$

$E(L) = (1+0.7)/0.604 +$ (100(0.606-0.3004+1.1169)+25+17)/(20+1.4225)

E(L)= 2.8148+184.25/21.4225 =11.415

For 2nd row:

$n = 7, h = 0.685, k = 3.04$

α=2(1-ϕ(3.04))=2(1-0.99882)=0.00236

β= $\phi(3.04 - 2\sqrt{7}) - \phi(-3.04 - 2\sqrt{7})$

$1 - \beta$=$\phi(2.25) \Rightarrow 0.98778$

$\lambda h = 0.685 * 0.05 = 0.03425$

$\tau = 0.3405$

$E(L) = (1+0.7)/0.685 +$ (100(0.6934-0.3405+1.1169)+25+3.386)/(20+1.4698)

$E(L) = 2.41 + 175.366 / 21.4698 = 10.649$

For 3rd row:

$n = 7, h = 0.766, k = 3.481$

α=2(1-ϕ(3.481))=2(1-0.9975)=0.0005

β= $\phi(3.481 - 2\sqrt{7}) - \phi(-3.481 - 2\sqrt{7})$

$1 - \beta = \phi(1.81) \Rightarrow 0.96485$

$\lambda h = 0.0383$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = (1 - (1.0383)0.96242) / 0.05(0.037575)$

$\tau = 0.3805$

$E(L) = (1 + 0.7) / 0.766$ + (100(0.7939-0.3805+1.1169)+25+0.6408)/(20+1.5303)

$E(L) = 10.351$

For 4th row:

$n = 8, h = 0.604, k = 3.04$

α=2(1-ϕ(3.04))

α=0.00236

β= $\phi(3.04 - 2\sqrt{8}) - \phi(-3.04 - 2\sqrt{8})$

$1 - \beta = \phi(2.62) \Rightarrow 0.9956$

$\lambda h = 0.604 * 0.05 = 0.0302$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = 0.3004$

$E(L) = (1 + 0.8) / 0.604$ + (100(0.6066-0.3004+1.1336)+25+3.848)/(20+1.4398)

$E(L) = 11.04$

For 5th row:

$n = 8, h = 0.685, k = 3.481$

α=2(1-ϕ(3.481))

α=0.0005

$\beta = \phi(3.481 - 2\sqrt{8}) - \phi(-3.481 - 2\sqrt{8})$

$1 - \beta = \phi(2.15) \Rightarrow 0.98537$

$\lambda h = 0.685 * 0.05 = 0.03425$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = 0.3405$

$E(L) = (1 + 0.8) / 0.685 +$ (100(0.695-0.3405+1.1336)+25+0.717)/(20+1.4881)

$E(L) = 10.74$

For 6th row:

$n = 8, h = 0.766, k = 2.599$

α=2(1-ϕ(2.6))

α=0.00932

$\beta = \phi(2.599 - 2\sqrt{8}) - \phi(-2.599 - 2\sqrt{8})$

$1 - \beta = \phi(3.05) \Rightarrow 0.99886$

$\lambda h = 0.766 * 0.05 = 0.0383$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = 0.3805$

$E(L) = (1 + 0.8) / 0.766 +$ (100(0.7668-0.3805+1.1336)+25+11.93)/(20+1.519)

$E(L) = 11.12$

For 7th row:

$n = 9, h = 0.604, k = 3.481$

α=2(1-ϕ(3.481))

α=0.0005

$\beta = \phi(3.481 - 2\sqrt{9}) - \phi(-3.481 - 2\sqrt{9})$

$1 - \beta = \phi(2.52) \Rightarrow 0.99413$

$\lambda h = 0.604 * 0.05 = 0.0302$

$\tau = (1 - (1 + \lambda h)e^{-\lambda h}) / \lambda(1 - e^{-\lambda h})$

$\tau = 0.3004$

$E(L) = (1 + 0.9) / 0.604 +$ (100(0.6416-0.3004+1.1503)+25+0.815)/(20+1.4915)

$E(L) = 11.286$

For 8th row:

$n = 9, h = 0.685, k = 2.599$

α=2(1-ϕ(2.6))

α=0.00932

β= $\phi(2.599 - 2\sqrt{9}) - \phi(-2.599 - 2\sqrt{9})$

$1 - \beta = \phi(3.4) \Rightarrow 0.99966$

$\lambda h = 0.685 * 0.05 = 0.03425$

$\tau = (1-(1+\lambda h)e^{-\lambda h}) / \lambda(1-e^{-\lambda h})$

$\tau = 0.3405$

$E(L) = (1+0.9)/0.685 +$ (100(0.6852-0.3405+1.1503)+25+13.373)/(20+1.4915)

$E(L) = 11.5103$

Table. 5.2. The optimum values of n,h,k of solved examples

Q.No	Journal & Author	a1	a2	a3	A3	a4	g	δ	D	E(L)	Our work (n,h,k)			Author (n,h,k)		
1.	A.Ganguly& S. K. Patel	1	0.1	25	50	100	0.0167	2	1	10.35	7	0.766	3.481	5	0.81	2.98
2.	Chao-yu chou, Chung- Ho-Chun	1	0.1	25	50	100	0.0167	2	1	10.35	7	0.766	3.481	2	0.59	2.9
3.	Achesom J. Duncan	0.5	0.1	25	50	100	0.05	2	2	4.347	5	0.755	3.775	46	22	2.3
4.	Tong-yuan-koo KENNETHE. E. CASE	20	20	1000	2000	4000	1.25	2	2	223.2	3	0.64	3.04	3	0.359	2.629

Furthermore, the results are compared with the optimum design of $\bar{X}$ control chart for the same example reported in Ganguly & S.K. Patel paper using simulated annealing, Chung- Ho Chen & Hui-Rong Liu paper using Grid search approach, Acheson J. Duncan and TONG-YUAN KOO & KENNETH E. CASE by using Direct Search method.

- From orthogonal arrays(OA)

  Sample size (n) = 7,

  Sampling interval (h) = 0.766 hr,

  Width of control limit (k) = 3.481

  Loss cost E(L) = 10.35$

  From simulated annealing (SA)

  Sample size (n) = 5,

  Sampling interval (h) = 0.81

  Width of control limit (k) = 2.98

  Loss cost E(L) = 10.37$

- From orthogonal arrays(OA)

  Sample size (n) = 7,

  Sampling interval (h) = 0.766hr,

  Width of control limit (k) = 3.481

  Loss cost E(L) = 10.35$

  From Grid Search Approach(GSA)

  Sample size (n) = 2

  Sample interval (h) = 0.59

  Width of control limit (k) = 2.9

  Loss cost E(L) = 14.435$

- From orthogonal arrays(OA)

  Sample size (n) = 5,

  Sampling interval (h) = 0.755,

  Width of control limit (k) =3.775

  Loss cost E(L) = 4.347$

  From J. Duncan

  Sample size (n) = 46

  Sample interval (h) = 22

  Width of control limit (k) = 2.3

  Loss cost E(L) = 94.97$

- From orthogonal arrays(OA)

  Sample size (n) = 3,

  Sampling interval (h) = 0.64,

  Width of control limit(k) = 3.04

  Loss cost E(L) = 223.2$

  From Direct search

  Sample size (n) = 3

  Sampling interval (h) = 0.359

  Width of control limit(k) = 2.62

  Loss cost E(L) = 242.58$

CHAPTER - 6

CONCLUSION

In this project, economical design for $\overline{X}$ control chart has been done by minimizing the loss cost using orthogonal arrays optimization algorithm illustrated by numerical examples. The result obtained are found to be superior to the reported earlier in literature.

Hence, Orthogonal arrays technique can be employed for design of $\overline{x}$ control charts.

Future Scope

- The preceding discussion has been confined to x chart but this technique can be applied to any type of control charts such as 'R' Chart, 'P' Chart, or 'C' Chart.
- In the present project, we have done economical design of x control chart but this technique can be applied to statistical design of control chart (or) statistical economical design of control charts.

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